16.50 Propulsion Systems
Year Unspecified

Quiz 1

One hour, open book, open notes

TRUE-FALSE QUESTIONS (50%)
Please include a 1-2 line explanation for each of your answers.

Statement	True	False
1. If one takes a given rocket, with a fixed chamber pressure, and replaces its operating gas by one with half the molecular mass, the thrust does not change. *$F = P_c A_t c_F$ and c_F depends on geometry and γ, not molecular mass.*	√	
2. For any external pressure, adding segments to a supersonic nozzle (increasing A_e/A_t) always increases thrust. *Maximum thrust when $P_e = P_a$. Extending the nozzle more than that creates suction, reduces thrust.*		√
3. A conventional rocket nozzle operates on the ground with $P_e = 0.5\ atm$. A device is proposed that will force flow separation at the point in the nozzle where $P = 1\ atm$. This will increase thrust. *Separation will allow back-filling of the negative pressure region past $P = 1\ atm$.*	√	
4. A small test rocket is fired in vacuum and measurements are made of the jet flow speed many exhaust diameters downstream. When this is done using different expansion area ratios A_e/A_t, the downstream speed is found to be invariant. *The speed for downstream is $c = F/\dot{m}$, i.e. the specific impulse (times g). This does vary with A_e/A_t.*		√
5. The method of characteristics can be used to design the contour of a nozzle, but only downstream from the throat. *The M.O.C. does require $M > 1$.*	√	
6. In a solid propellant rocket, increasing the throat area increases the thrust. *$F = P_c A_t c_F;\ P_c = \left(\rho_p a c^* \frac{A_b}{A_t}\right)^{\frac{1}{1-n}}$, so $F \sim {A_t}^{1-\frac{1}{1-n}} = {A_t}^{\frac{-n}{1-n}}$, F decreases with A_t. In addition, c_F also decreases (through $\frac{A_e}{A_t}$ decreasing).*		√
7. The characteristic damping time of pressure oscillations inside a solid propellant rocket is proportional to the linear dimensions (assuming geometrically similar rockets). *$\tau \sim (1-n)\frac{V_{ch}\rho_c}{\dot{m}} = (1-n)\frac{V_c \rho_c}{P_c A_t} c^* \sim \frac{V_c}{A_t}$ For similar geometry, $\frac{V_c}{A_t} \sim L_c$*	√	
8. In an equilibrium nozzle expansion, dissociated species recombine fairly completely. This means the performance is the same as one would calculate if dissociation were ignored in the chamber. *The heat of dissociation is all recovered in the nozzle recombination, but part of it at $P < P_c$, so the expansion to P_e is less efficient than it would be from P_c.*		√
9. The chemical reactions that are selected for imposing equilibrium in the calculation of chamber temperature must be the ones that actually happen during combustion. *They can be selected arbitrarily, as long as they linearly span the space of possible reactions.*		√

10. Heat flux to the nozzle walls peaks at the throat because that is where stagnation temperature is maximum. *Stagnation temperature is T_c everywhere. What is maximum at the throat is ρu.*		√

PROBLEM (50%)

The gas leaving the combustion chamber of a LOX-LH rocket has the following characteristics:

- Molecular mass: $M = 13\ g/mol$
- Specific heat ratio: $\gamma = 1.26$
- Temperature: $T_c = 3600\ K$
- Pressure: $P_c = 210\ atm$
- Viscosity: $\mu_g = 3 \times 10^{-5} \left(\frac{T}{3000}\right)^{0.6}\ Kg/m/s$
- Prandtl's number: $P_r = 0.9$

The nozzle throat has a diameter $D_t = 0.277\ m$, and its first wall is a thin Copper shell which is cooled on its back side to $T_{wc} = 300\ K$. The thermal conductivity of Copper is $k = 360\ W/m/K$.

What is the maximum Copper thickness such that the hot-side temperature T_{wh} does not exceed $800\ K$?

Formulas and constants:

$$\rho_t u_t = \frac{P_c}{c^*},\ with\ c^* = \frac{\sqrt{RT_c}}{\Gamma(\gamma)},\ \Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\Gamma = \sqrt{1.26}\left(\frac{2}{2.26}\right)^{\frac{2.26}{2\times 0.26}} = 0.6599$$

$$R = \frac{8.314}{0.013} = 639.5\ {}^{J}/_{kgK}$$

To calculate:

$$c^* = 2299\frac{m}{s} \rightarrow \rho_t u_t = \frac{210 \times 1.013 \times 10^5}{2299} = 9252\ {}^{kg}/_{m^2 s}$$

$$T_t = \frac{T_c}{1 + \frac{\gamma-1}{2} \times 1^2} = 3186\ K \rightarrow \mu_t = 3 \times 10^{-5}\left(\frac{3186}{3000}\right)^{0.6} = 3.11 \times 10^{-5}\ {}^{kg}/_{m/s}$$

Reynolds number:

$$Re_t = \frac{\rho_t u_t D_t}{\mu_t} = \frac{9252 \times 0.277}{3.11 \times 10^{-5}} = 8.24 \times 10^7$$

$$c_f \approx \frac{0.046}{Re_t^{0.2}} = \frac{0.046}{(1.24 \times 10^7)^{0.2}} = 1.201 \times 10^{-3}$$

Stanton number:

$$S_t = \frac{c_f}{2P_r^{\ 0.67}} = \frac{1.201 \times 10^{-3}}{2 \times 0.9^{0.67}} = 6.44 \times 10^{-4}$$

Heat flux at throat:

$$q_w = \rho_t u_t c_p (T_c - T_{wh}) S_t = 9252\left(\frac{1.26}{0.26} \times 639.5\right)(3600 - 800) \times 6.44 \times 10^{-4}$$
$$q_w = 5.17 \times 10^7 W/m^2$$

Heat conduction through Copper shell:

$$q_w = h\frac{T_{wh} - T_{wc}}{\delta_{cu}}$$
$$\delta_{cu} = k\frac{T_{wh} - T_{wc}}{q_w} = 360\frac{800 - 300}{5.17 \times 10^7} = 3.48 \times 10^{-3} W/m^2$$

So the shell must be less than 3.5 mm thick.

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