Image credit: NASA.

Notation

- S^{t+1} set of hidden variables in the t+1 time slice
- s^{t+1} set of values for those hidden variables at t+1
- o^{t+1} **set of observations at time t+1**
- $o^{1:t}$ **set of observations from all times from 1 to t**
- α **normalization constant**

10/28/07 2

Multiple Faults Occur

Image source: NASA. APOLLO 13

- three shorts, tank-line and pressure jacket burst, panel flies off.

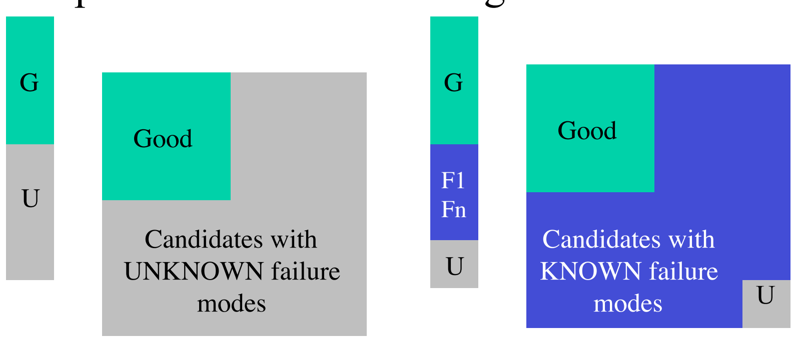
Lecture 12: Framed as CSP.

- How do we compare the space of alternative diagnoses?
- How do we prefer diagnoses that explain failure?

3

10/28/07

Due to the unknown mode, there tends to be an exponential number of diagnoses.



1. Introduce fault models.
   - More constraining, hence more easy to rule out.
   - Increases size of candidate space.
2. Enumerate most likely diagnoses X_i based on probability.
   Prefix (k) (Sort $\{X_i\}$ by decreasing $P(X_i \mid O)$)
   - Most of the probability mass is covered by a few diagnoses.

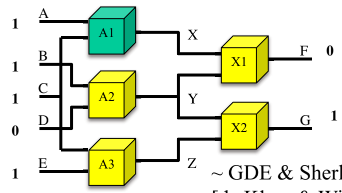
4

10/26/10

Model-based Diagnosis

Xor(i):

- G(i):
  Out(i) = In1(i) xor In2(i)
- Stuck_0(i):
  Out(i) = 0
- U(i):



~ GDE & Sherlock
[de Kleer & Williams, 87, 89]

Input:

- Finite Domain Variables
  - <X,Y>
    - X — mode variables
    - Y — model variables
  - O — observable variables $O \subseteq Y$.
- $\Phi(X, Y)$ — model constraints
- o — observations $o_{i:n} \in D_O$
- $P(X_i)$ — a prior probability of modes

Output:

- $\{x \in D_X \mid \exists y \in D_Y$ s.t. $x \wedge o \wedge \Phi(x,y)$ is consistent$\}$

$\Rightarrow$ $P(X \mid o_{i:n})$

Assumptions:

- Modes are static
- uniform dist on logical models.
- $X_i \perp X_j$ for $i \neq j$ *(apriori)*
- $O_i \perp O_j \mid X$ for $i \neq j$

Consistency-based

Probabilistic,
Sequential Observations

Compute Conditional Probability via Bayes Rule (Method 6)

$$P(X \mid o) = \frac{P(o \mid X)P(X)}{P(o)} \qquad = \alpha P(o \mid X)P(X)$$

$$= \frac{P(o \mid X)P(X)}{\sum_{x \in X} P(o,x)}$$

$$= \frac{P(o \mid X)P(X)}{\sum_{x \in X} P(o \mid x)P(x)}$$

Candidate Prior Probabilities

$$P(X) = \prod_i P(X_i \mid X_{1:i-1})$$ Chain rule

Assume $X_i \perp X_j$ for $i \neq j$:

$$P(X) = \prod_{X_i \in X} P(X_i)$$

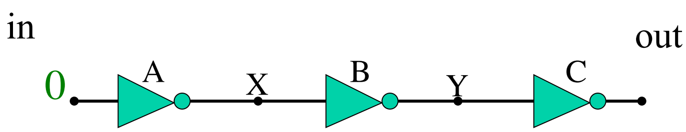
	A	B	C
P(G)	.99	.99	.99
P(S1)	.008	.008	.001
P(S0)	.001	.001	.008
P(U)	.001	.001	.001

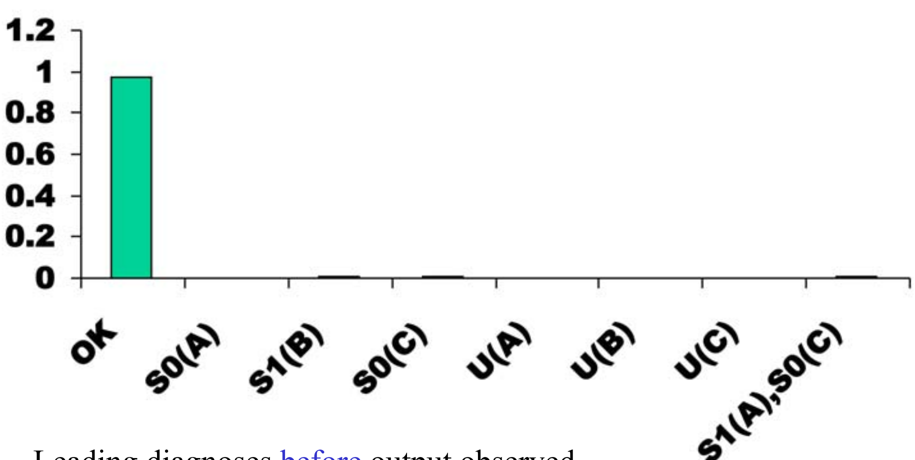
$P(A = G, B = G, C = G) = .97$

$P(A = S1, B = G, C = G) = .008$

$P(A = S1, B = G, C = S0) = .00006$

$P(A = S1, B = S1, C = S0) = .0000005$





Leading diagnoses before output observed

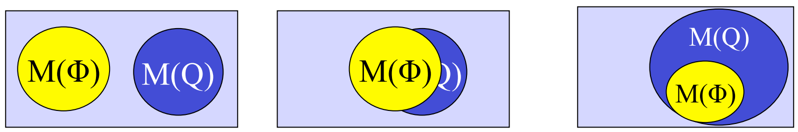
Estimate probability by assuming a uniform distribution (Method 7)

$$P(X \mid o) = \alpha P(o \mid X) P(X)$$

How do we compute P(o | X), from logical formulae Φ(X,Y)?

Given theory Φ, sentence Q ….

… is inconsistent. … could be true. … must be true.



P(Q | Φ)= 0 $\frac{P(Q,\Phi)}{P(\Phi)} = \frac{\sum_{s_i \in Q \cap \Phi} P(s_i)}{\sum_{s_i \in \Phi} P(s_i)}$ 1

Problem: Logic doesn't specify $P(s_i)$ for models of consistent sentences.

10/28/07 9

Estimate probability by assuming a uniform distribution (Method 7)

Problem: Logic doesn't specify $P(s_i)$ for models of consistent sentences.
⇒ Assume all models are equally likely → count models.

$$P(Q \mid \Phi) = \frac{P(Q,\Phi)}{P(\Phi)} = \frac{\sum_{s_i \in Q \cap \Phi} P(s_i)}{\sum_{s_i \in \Phi} P(s_i)} = \frac{|M(Q \wedge \Phi)|}{|M(\Phi)|}$$

Model-based Diagnosis using model counting:

$$P(o \mid x) = \frac{P(o,x)}{P(x)} = \frac{|M(o \wedge x \wedge \Phi(x,Y))|}{|M(x \wedge \Phi(x,Y))|}$$

10/28/07 10

Simplify P(O | M) using the Naïve Bayes Assumption (Method 7)

$$P(X \mid o) = \alpha P(o \mid X) P(X)$$

Problem: P(o | X) can be hard to determine for large |o|.

Assume: observations o are independent given mode X.

$$o_1 \perp o_2 \ldots \perp o_n \mid X$$

$$P(o \mid X) = P(o_1 \mid X) P(o_2 \mid X) \ldots P(o_n \mid X)$$

by Naïve Bayes

Diagnosis with Sequential Observations and the Naïve Bayes Assumption

$$P(X \mid o_{1:n}) = \frac{P(o_n \mid X, o_{1:n-1}) P(X \mid o_{1:n-1})}{P(o_n \mid o_{1:n-1})} \quad = \alpha P(o_n \mid X, o_{1:n-1}) P(X \mid o_{1:n-1})$$

$$= \frac{P(o_n \mid X, o_{1:n-1}) P(X \mid o_{1:n-1})}{\sum_{x \in X} P(o_n, x \mid o_{1:n-1})}$$

$$= \frac{P(o_n \mid X, o_{1:n-1}) P(X \mid o_{1:n-1})}{\sum_{x \in X} P(o_n \mid x, o_{1:n-1}) P(x \mid o_{1:n-1})}$$

Assume: observations o are independent given mode X.

$$o_1 \perp o_2 \ldots \perp o_n \mid X$$

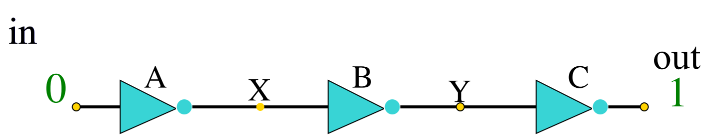
$$= \frac{P(o_n \mid X) P(X \mid o_{1:n-1})}{\sum_{x \in X} P(o_n \mid x) P(x \mid o_{1:n-1})}$$

Estimating the Observation Probability $P(o_i \mid M, o_{1:n-1})$ in GDE

GDE used naïve Bayes AND assumed consistent observations for candidate m are equally likely.

$P(o_n \mid x, o_{1:n-1})$ is estimated using model $\Phi(x,Y)$ according to:

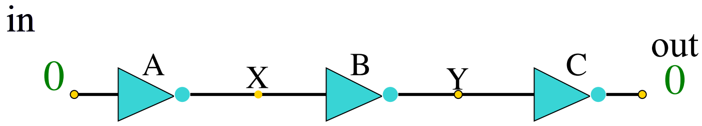
- **If** $o_{1:n-1} \wedge x \wedge \Phi(x,Y)$ entails o_n
  Then $P(o_n \mid x, o_{1:n-1}) = 1$
- **If** $o_{1:ni-1} \wedge x \wedge \Phi(x,Y)$ entails $O_n \neq o_n$
  Then $P(o_n \mid m, o_{1:n-1}) = 0$
- **Otherwise**, Assume all consistent assignments to O_n are equally likely:
  let $D_{Cn} \equiv \{o_c \in D_{On} \mid o_{1:n-1} \wedge x \wedge \Phi(x,Y)$ is consistent with $O_n = o_c\}$
  Then $P(o_n \mid x, o_{1:n-1}) = 1 / |D_{Cn}|$



$$P(X \mid o_{1:n}) = \alpha P(o_n \mid X) P(X \mid o_{1:n-1})$$

Observe out = 1:

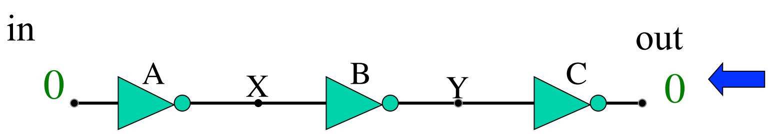
- x = <A=G, B=G, C=G>
- Prior: P(x) = .97
- P(out = 0 | x) = 1
- P(x | out = 0) = α x 1 x .97 = .97



$$P(X \mid o_{1:n}) = \alpha P(o_n \mid X) P(X \mid o_{1:n-1})$$

Observe out = 0:

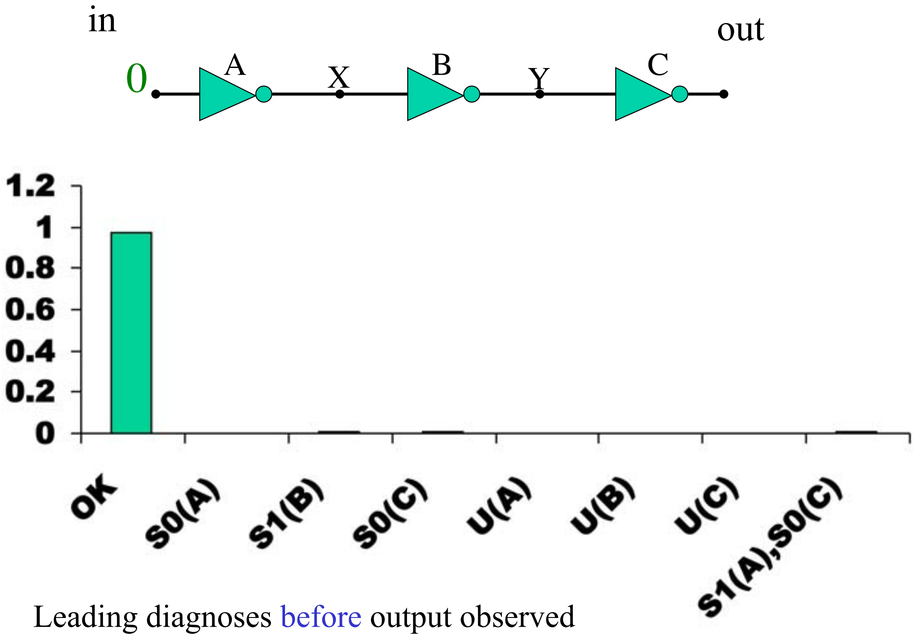
- x = <A=G, B=G, C=G>
- Prior: P(x) = .97
- P(out = 0 | x) = 0
- P(x | out = 0) = 0 x .97 x α = 0



Priors for Single Fault Diagnoses:

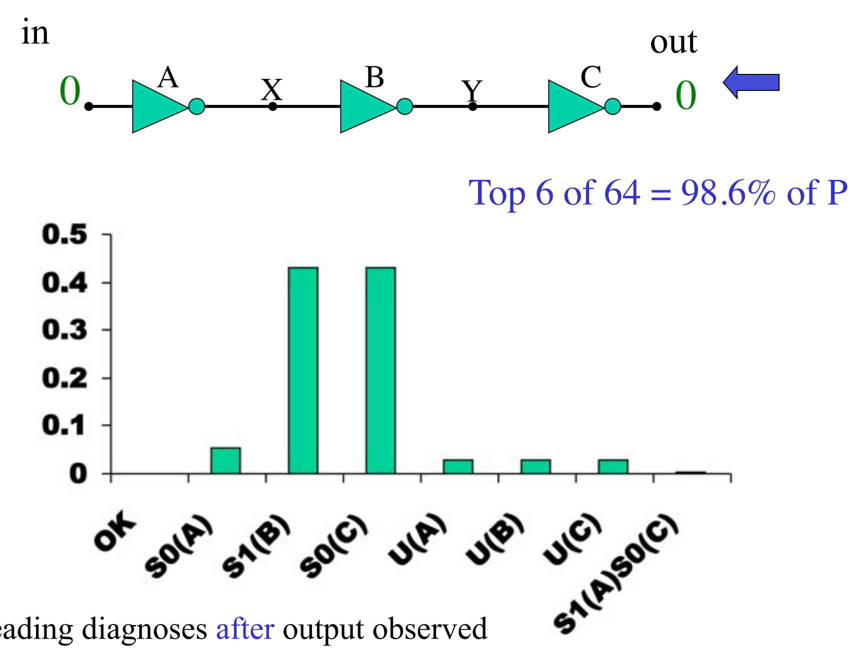
	A	B	C
P(S1)	~~.008~~	.008	~~.001~~
P(S0)	.001	~~.001~~	.008
P(U)	.001	.001	.001

10/26/10 16



Leading diagnoses before output observed

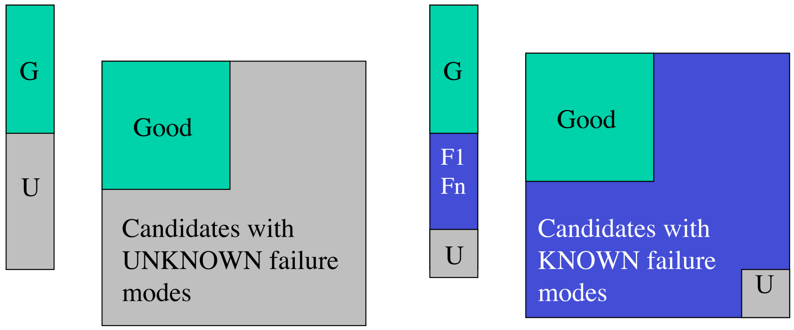
10/26/10 17



Leading diagnoses after output observed

10/26/10 18

Due to the unknown mode, there tends to be an exponential number of diagnoses.

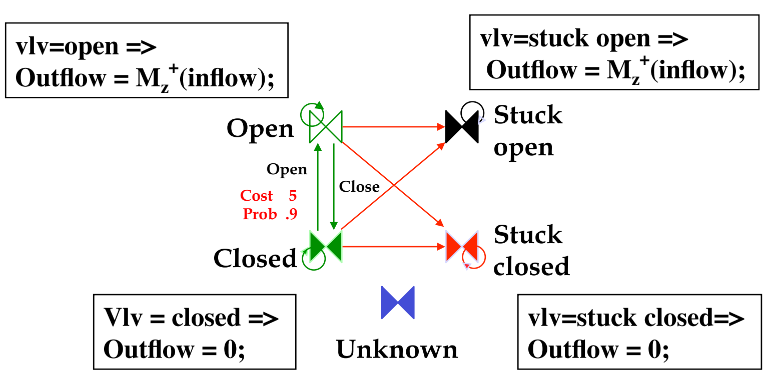


But "unknown" diagnoses represent a small fraction of the probability density space.

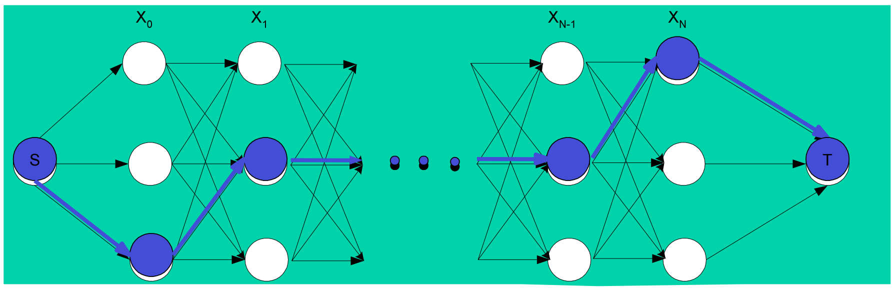
Most of the density space may be approximated by enumerating the few most likely diagnoses.

Diagnosing Dynamic Systems: Probabilistic Constraint Automata

Probabilistic transitions between modes



Estimating Dynamic Systems



Given sequence of commands and observations:

- Infer current distribution of (most likely) states.
- Infer most likely trajectories.