

Problem Solving as State Space Search

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16.410-13

Sept 13th, 2010

Slides adapted from:
6.034 Tomas Lozano Perez,
Russell and Norvig AIMA

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Assignments

- **Remember:**
Problem Set #1: Java warm up
Out last Wednesday,
Due this Wednesday, September 15th
- **Reading:**
 - Today: Solving problems through search [*AIMA*] Ch. 3.1-4
 - Wednesday: Asymptotic Analysis Lecture 2 Notes of 6.046J;
Recurrences, Lecture 12 Notes of 6.042J.

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Recap - Course Objectives

1. Understand the major types of agents and architectures:
 - goal-directed vs utility-based
 - deliberative vs reactive
 - model-based vs model-free
2. Learn the modeling and algorithmic building blocks for creating agents:
 - Model problem in an appropriate formal representation.
 - Specify, analyze, apply and implement reasoning algorithms to solve the problem formulations.

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Recap – Agent Architectures



Functions: Robust, coordinated operation + mobility

It Begins with State Space Search!

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Problem Solving as State Space Search

- Problem Formulation (Modeling)
 - Problem solving as state space search
- Formal Representation
 - Graphs and search trees
- Reasoning Algorithms
 - Depth and breadth-first search

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Most Agent Building Block Implementations Use Search

Robust Operations:

- Activity Planning
- Diagnosis
- Repair
- Scheduling
- Resource Allocation

Mobility:

- Path Planning
- Localization
- Map Building
- Control Trajectory
Design

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Example: Outpost Logistics Planning



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Image produced for NASA by John Frassanito and Associates.

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Can the astronaut get its supplies safely across a Lunar crevasse?

Astronaut
Goose
Grain
Fox

- Astronaut + 1 item allowed in the rover.
- Goose alone eats Grain
- Fox alone eats Goose

Rover 

Early AI: What are the universal problem solving methods?

Simple  Trivial

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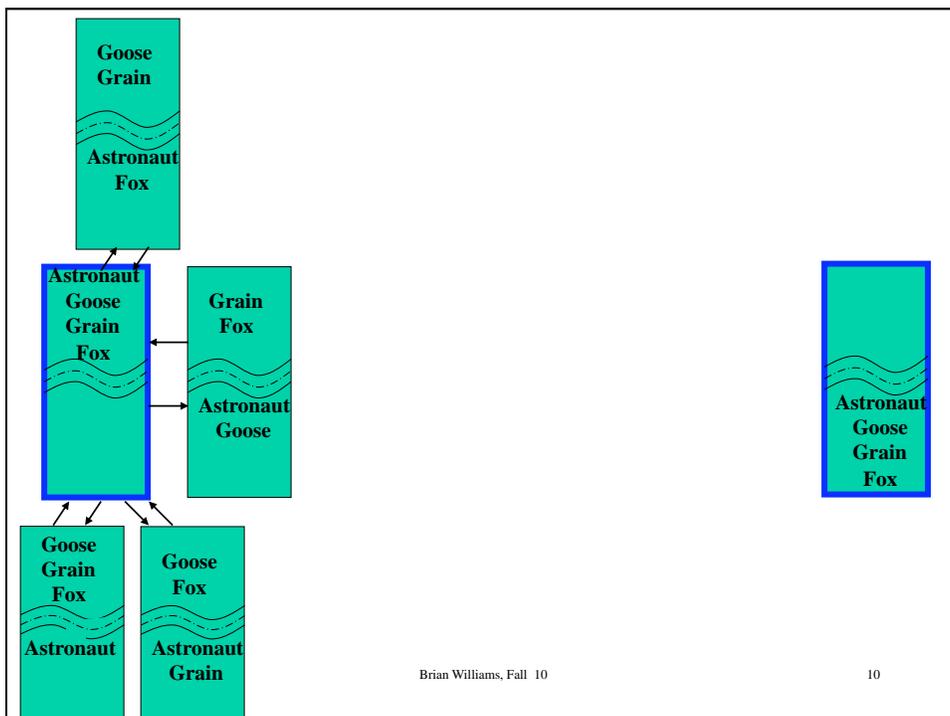
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Problem Solving as State Space Search

- Formulate Goal
 - State
 - Astronaut, Fox, Goose & Grain below crevasse.
- Formulate Problem
 - States
 - Astronaut, Fox, Goose & Grain above or below the crevasse.
 - Operators
 - Move: Astronaut drives rover and 1 or 0 items to other side of crevasse.
 - Initial State
 - Astronaut, Fox, Goose & Grain above crevasse.
- Generate Solution
 - Sequence of Operators (or States)
 - Move(goose,astronaut), Move(astronaut), . . .

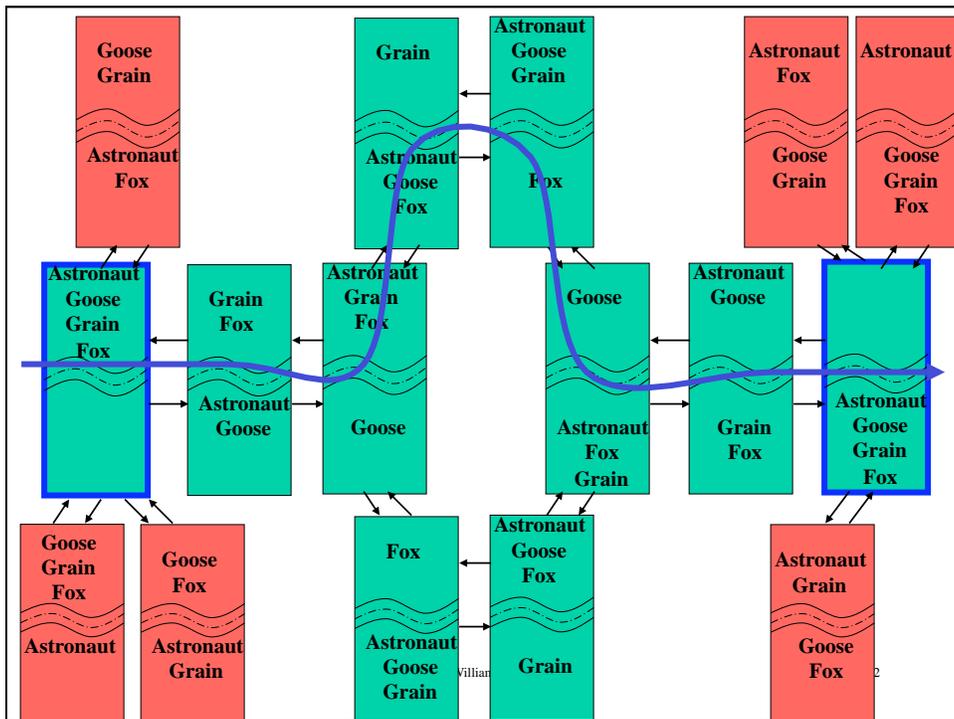
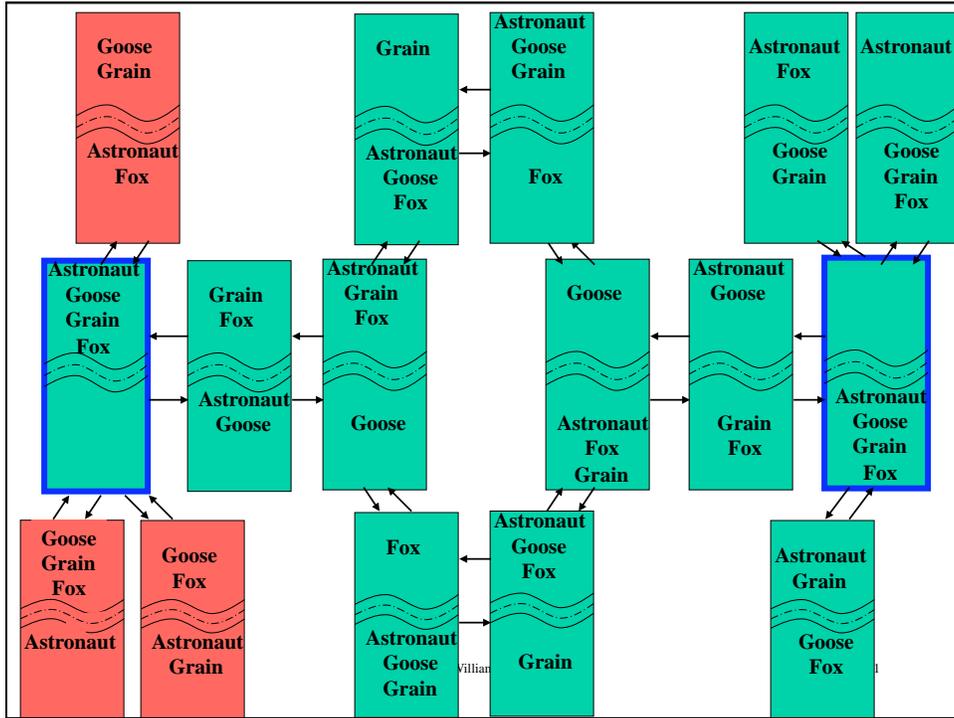
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Formulation Example: 8-Puzzle

5	4	
6	1	8
7	3	2

Start

1	2	3
8		4
7	6	5

Goal

- States: integer location for each tile AND ...??
- Operators: move empty square up, down, left, right
- Initial and Goal States: as shown above

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Languages for Expressing States and Operators for Complex Tasks



- Swaggert & Lovell assemble emergency rig for Apollo 13 lithium hydroxide unit.

Image source: NASA.

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Example: STRIPS Planning Language

Initial state:

(and (hose a)
 (clamp b)
 (hydroxide-unit c)
 (on-table a)
 (on-table b)
 (on-table c)
 (clear a)
 (clear b)
 (clear c)
 (empty arm))

goal (partial state):

(and (connected a b)
 (connected b c)))

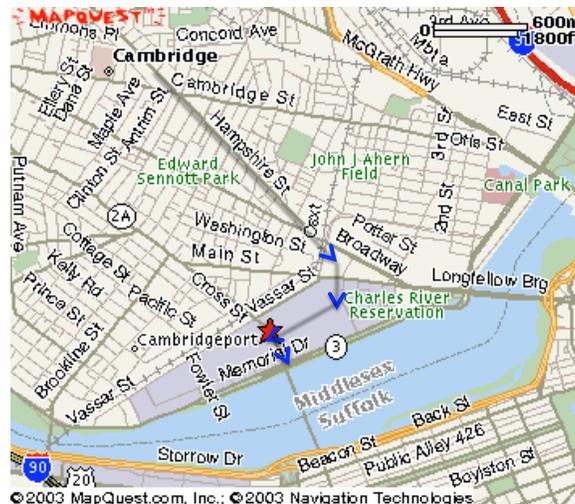
Operators

precondition: (and (clear hose)
 (on-table hose)
 (empty arm))

pickup hose

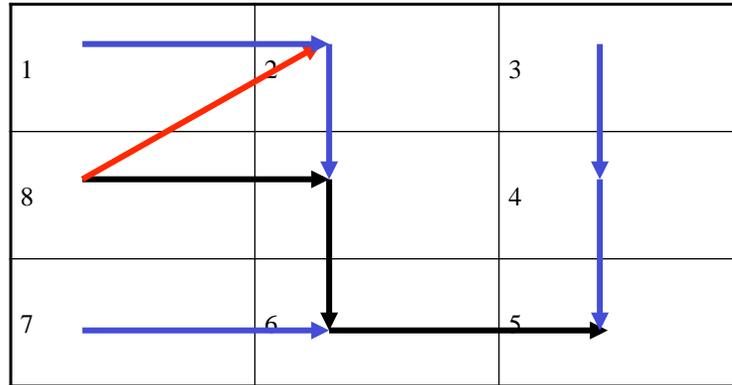
effect: (and (not (clear hose))
 (not (on-table hose))
 (not (empty arm))
 (holding arm hose)))

Problem: Find a Route from home to MIT



States? Operators?, Initial and Goal State?

Problem: Compensating for Error Online

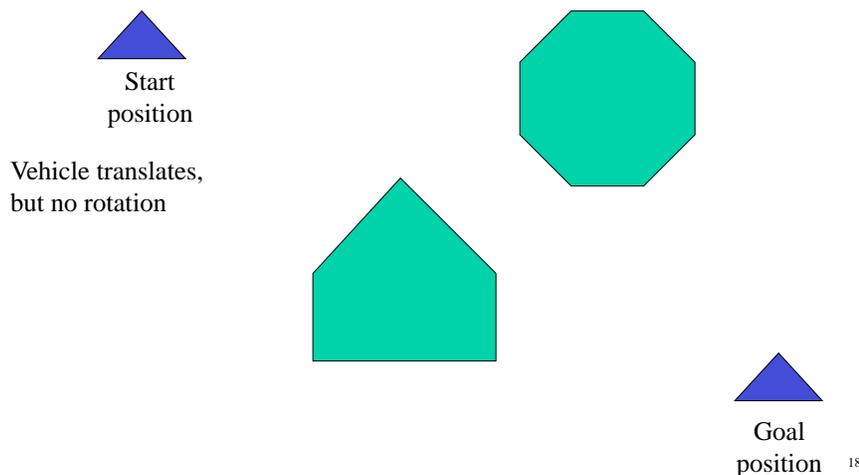


- Policy, $\pi(v) \rightarrow e$, dictates how to act in all states.
- Policy π corresponds to a shortest path tree from **all vertices** to the **destination**.

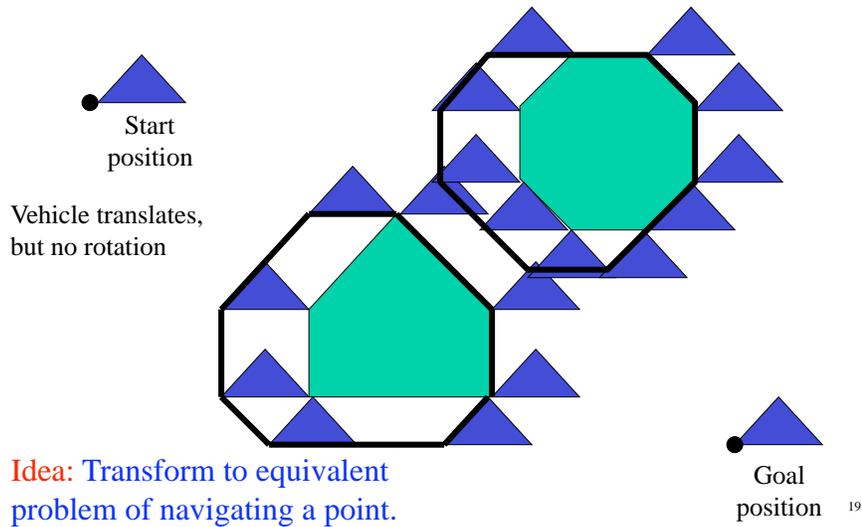
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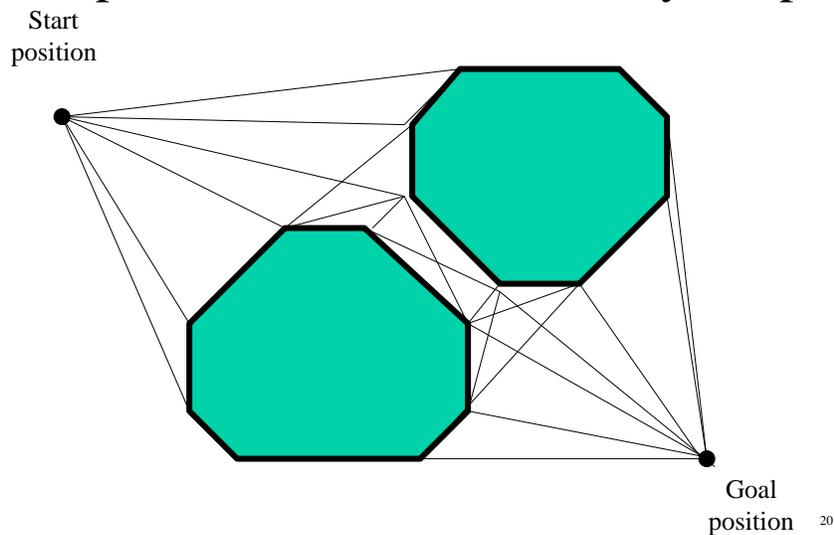
How do we Map Path Planning to State Space Search?



1. Create Configuration Space

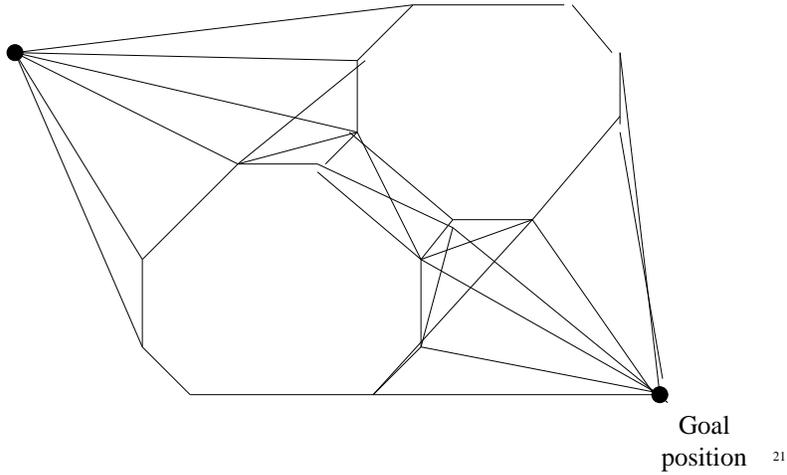


2. Map From Continuous Problem to Graph Search: Create Visibility Graph



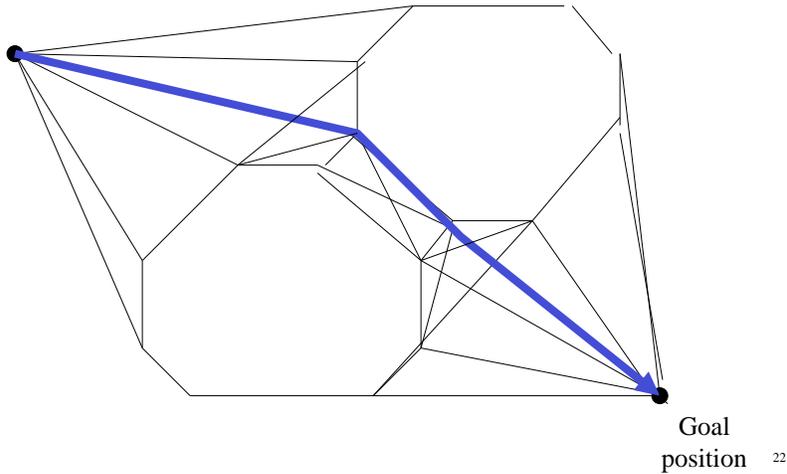
2. Map From Continuous Problem to Graph Search: Create Visibility Graph

Start
position

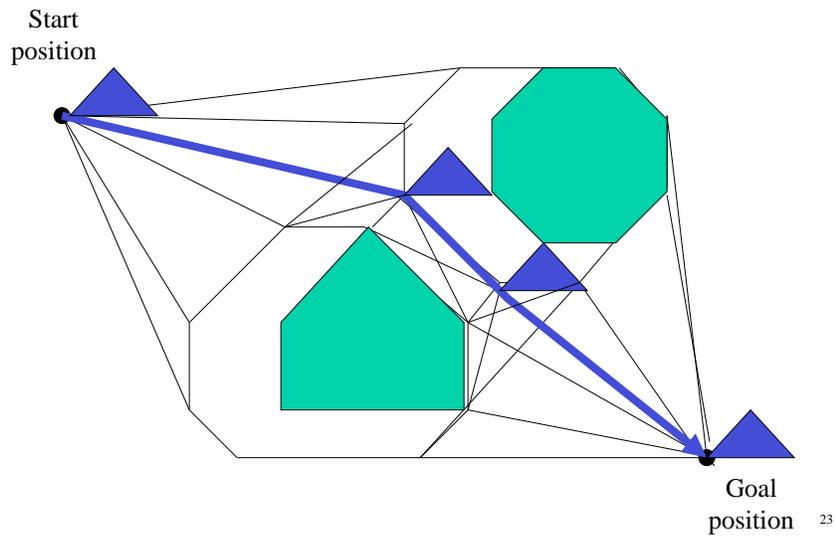


3. Find Shortest Path (e.g., A*)

Start
position



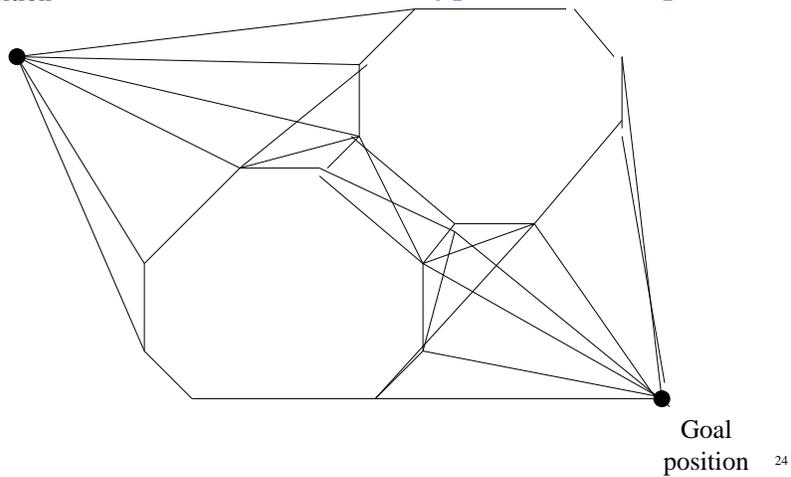
Resulting Solution



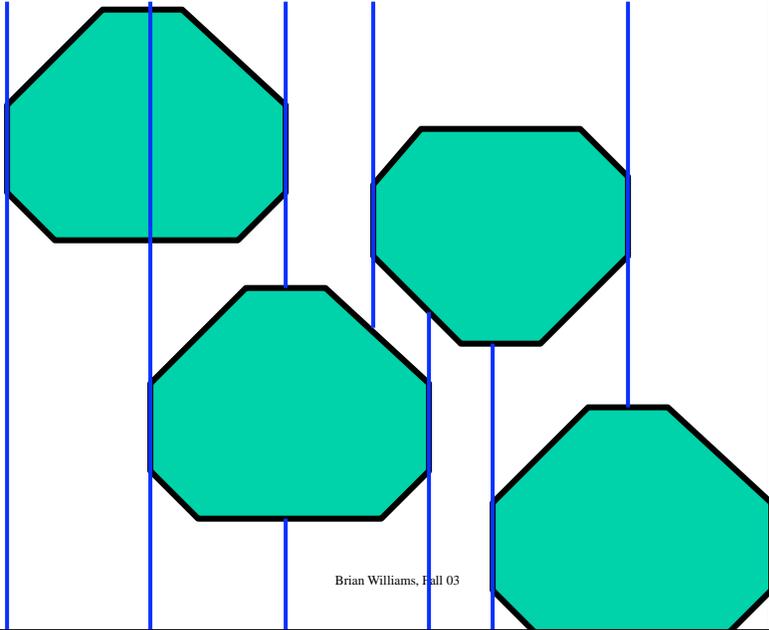
A Visibility Graph is a Kind of Roadmap

Start position

What are the strengths / weaknesses of roadmaps?
What are some other types of roadmaps?



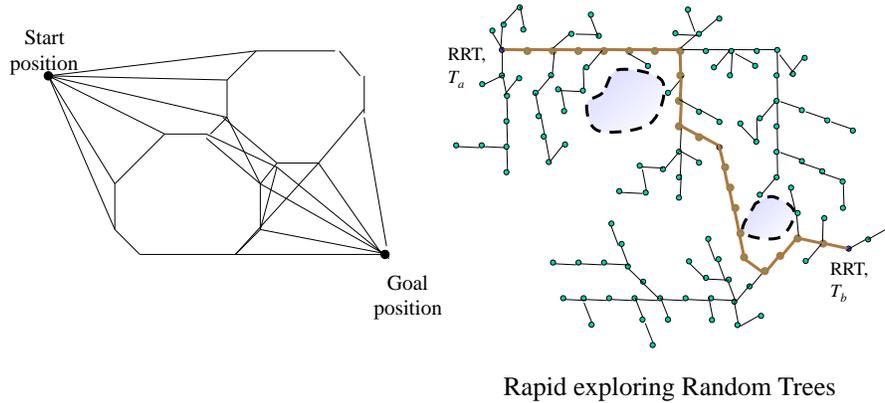
Roadmaps: *Exact* Cell Decomposition



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How do we handle large state spaces?



Rapid exploring Random Trees

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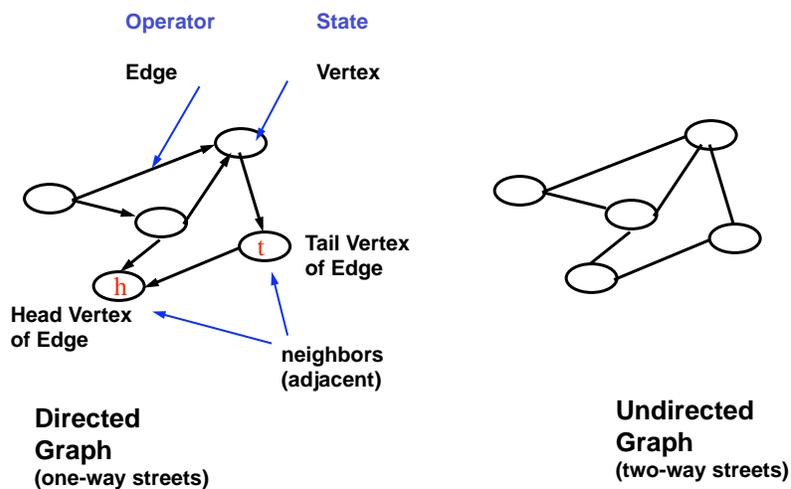
Problem Solving as State Space Search

- Problem Formulation (Modeling)
 - Problem solving as state space search
- Formal Representation
 - Graphs and search trees
- Reasoning Algorithms
 - Depth and breadth-first search

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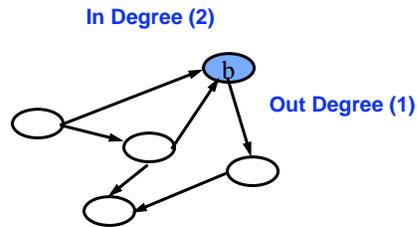
Problem Formulation: A Graph



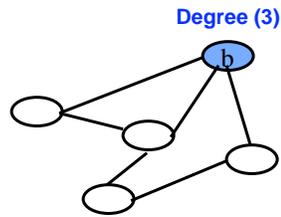
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Problem Formulation: A Graph



Directed Graph
(one-way streets)



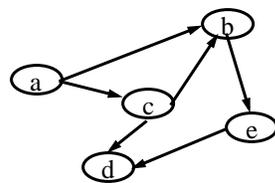
Undirected Graph
(two-way streets)

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Problem Formulation: A Graph

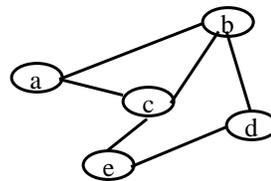
Strongly connected graph
Directed path between all vertices.



Directed Graph
(one-way streets)

Connected graph
Path between all vertices.

Complete graph
All vertices are adjacent.



Undirected Graph
(two-way streets)

Sub graph
Subset of vertices
edges between vertices in Subset

Clique
A complete subgraph
(All vertices are adjacent).

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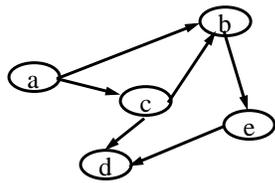
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Specifying a Graph: $G = \langle V, E \rangle$

Notation:

$\langle a, b, \dots n \rangle$ an ordered **list** of elements $a, b \dots$

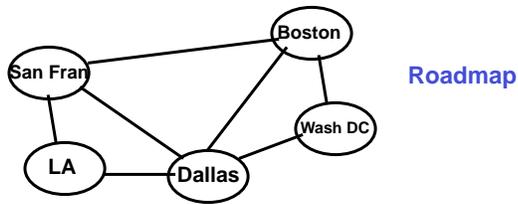
$\{a, b, \dots n\}$ an unordered **set** of **distinguished** elements.



Vertices $V = \{a, b, c, d, e\}$

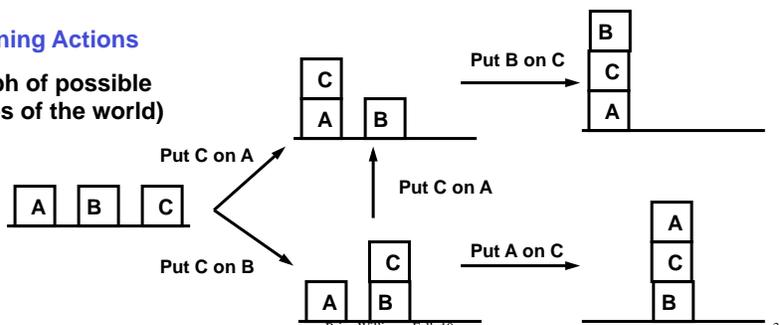
Edges $E = \{\langle a, b \rangle, \langle a, c \rangle, \langle b, e \rangle, \langle c, b \rangle, \langle c, d \rangle, \langle e, d \rangle\}$

Examples of Graphs



Planning Actions

(graph of possible states of the world)

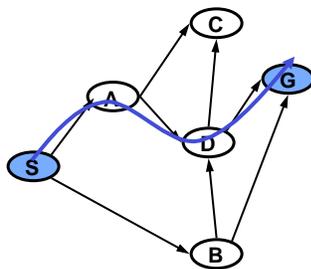


Formalizing State Space Search

Input: A search problem $S = \langle g, S, G \rangle$ where

- graph $g = \langle V, E \rangle$,
- start vertex S in V , and
- goal vertex G in V .

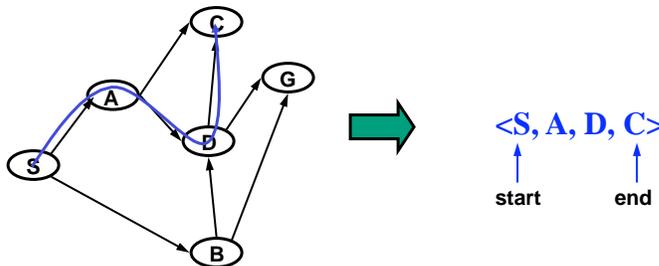
Output: A simple path $P = \langle S, v_2, \dots, G \rangle$ in g from S to G .



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Simple Paths of Graph $g = \langle V, E \rangle$



A (directed) path P of graph g is

a sequence of vertices $\langle v_1, \dots, v_n \rangle$ in V
such that each successive pair $\langle v_i, v_{i+1} \rangle$ is a (directed) edge in E

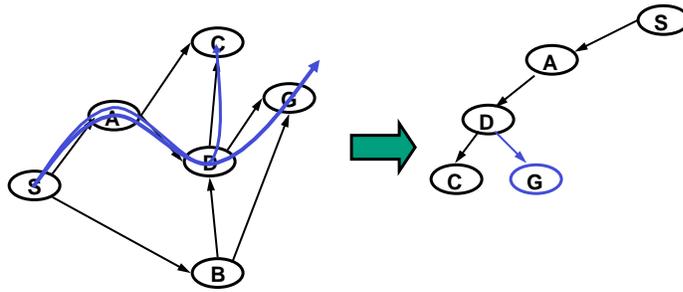
A simple path is a path that has no cycles.

A cycle is a subpath where start = end (*i.e.*, repeated vertices).

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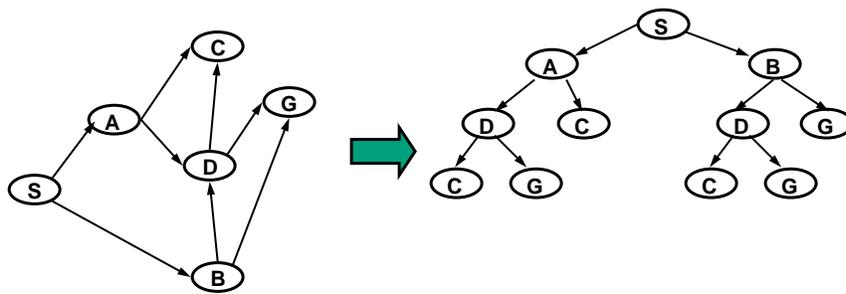
A Problem Solver Searches through all Simple Paths



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A Search Tree Denotes All Simple Paths



Enumeration is:

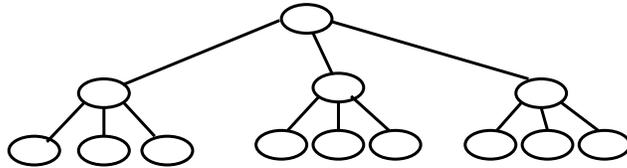
- Complete
- Systematic
- Sound

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Search Trees

think of a **tree** as a “family” tree



A **tree T** is a **directed graph**, such that

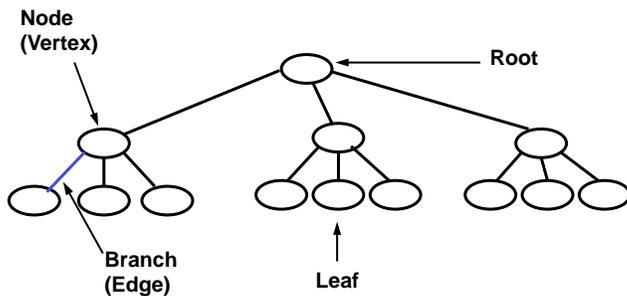
- there exists exactly **one undirected path** between any **pair of vertices**.

- **In degree** of each vertex is **1**

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Search Trees

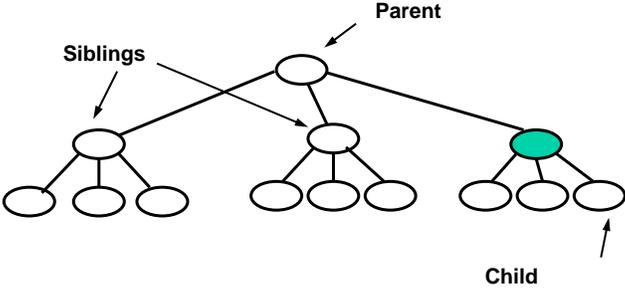


think of a **tree** as a “family” tree

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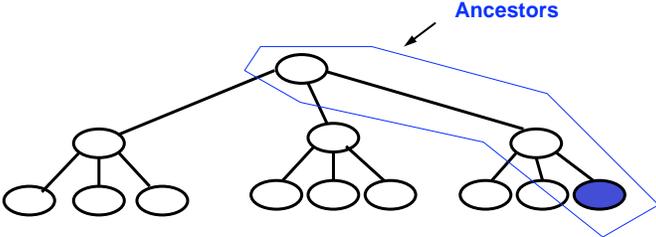
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Search Trees



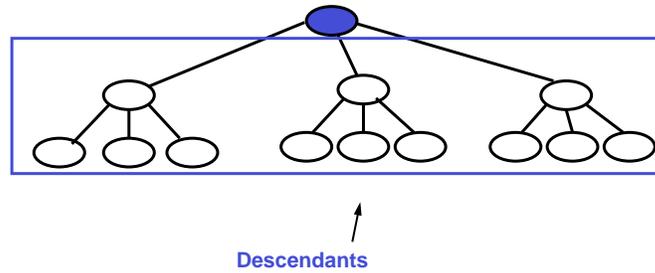
think of a tree as a “family” tree

Search Trees



think of a tree as a “family” tree

Search Trees



think of a [tree](#) as a “family” tree

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Problem Solving as State Space Search

- Problem Formulation (Modeling)
 - Problem solving as state space search
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Classes of Search

Blind (uninformed)	Depth-First	Systematic exploration of whole tree until the goal is found.
	Breadth-First	
	Iterative-Deepening	

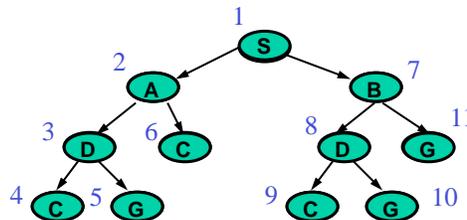
Optimal (informed)	A*	Use path "length" measure. Find "shortest" path.
	Branch&Bound	

Heuristic (informed)	Hill-Climbing	Use heuristic measure of goodness of a node.
	Best-First	
	Beam	

Depth First Search (DFS)

Local Rule: After visiting node...

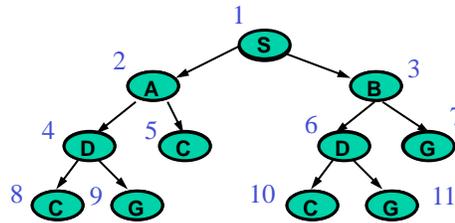
- Visit its **children** before its **siblings**
- Visit its **children left to right**



Breadth First Search (BFS)

Local Rule: After visiting node...

- Visit its **siblings**, before its **children**
- Visit its **children left to right**



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Elements of Algorithm Design

Algorithm Description: (Today)

- **stylized pseudo code**, sufficient to analyze and implement the algorithm (implementation next Wednesday).

Algorithm Analysis: (Wednesday & Monday)

- **Time complexity:**
 - how long does it take to find a solution?
- **Space complexity:**
 - how much memory does it need to perform search?
- **Soundness:**
 - when a solution is returned, is it guaranteed to be correct?
- **Completeness:**
 - is the algorithm guaranteed to find a solution when there is one?

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Problem Solving as State Space Search

- Problem Formulation (Modeling)
- Formal Representation
- Reasoning Algorithms
 - A generic search algorithm description
 - Depth-first search example
 - Handling cycles
 - Breadth-first search example

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Solve $\langle g = \langle V, E \rangle, S, G \rangle$ using State Space Search

Search States:

- All simple paths $\langle S, \dots v \rangle$ in g starting at S

Initial State:

- $\langle S \rangle$

Operator:

- Extend a path $\langle S, \dots v \rangle$ to $\langle S, \dots v, u \rangle$
for each $\langle v, u \rangle$ in E
 - call u a **child** of v

Goal:

- A simple path $\langle S, \dots, G \rangle$ in g

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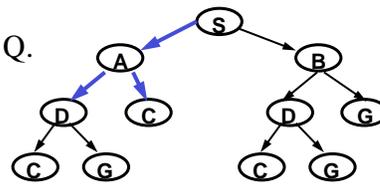
Solve $\langle g = \langle V, E \rangle, S, G \rangle$ using State Space Search

How do we maintain the search state?

- An ordering on partial paths yet to be expanded (called a **queue Q**).

How do we perform search?

- Repeatedly:
 1. Select next partial path from Q.
 2. Expand it.
 3. Add expansions to Q.
- Terminate when goal G is found.

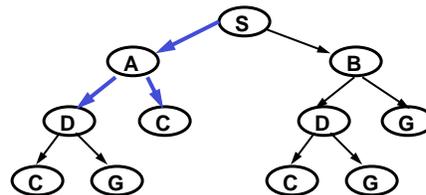


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Simple Search Algorithm: Preliminaries

- A **partial path** from S to D is listed in **reverse** order,
 - e.g., $\langle D, A, S \rangle$
- The **head** of a partial path is its **most recent** visited node,
 - e.g., D.
- The **Q** is a **list** of partial paths,
 - e.g. $\langle \langle D, A, S \rangle, \langle C, A, S \rangle \dots \rangle$.



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Simple Search Algorithm

Let Q be a list of partial paths,
S be the Start node and
G be the Goal node.

1. Initialize Q with partial path <S>
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
 - a) Remove N from Q
 - b) Find all children of head(N) and create a one-step extension of N to each child
 - c) Add all extended paths to Q
 - d) Go to step 2

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Problem Solving as State Space Search

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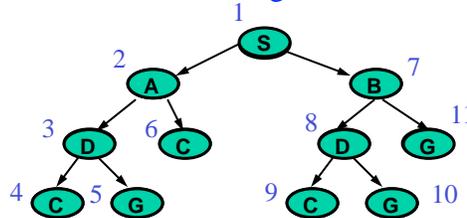
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Depth First Search (DFS)

Idea: After visiting node

- Visit its **children** left to right (or top to bottom)
- Visit its **children** before its **siblings**



Assume we remove the **first element** of Q,

Where to Q do we **add** the path extensions?

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Simple Search Algorithm

Let Q be a list of partial paths,
S be the start node and
G be the Goal node.

1. **Initialize Q with partial path <S>**
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N **(goal reached!)**
4. Else:
 - a) Remove N from Q
 - b) Find all children of head(N) and create a one-step extension of N to each child
 - c) Add all extended paths to Q
 - d) Go to step 2

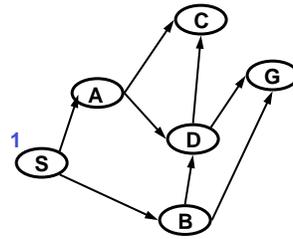
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Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	
3	
4	
5	



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Simple Search Algorithm

Let Q be a list of partial paths,
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G be the Goal node.

1. Initialize Q with partial path <S>
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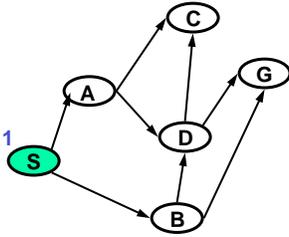
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Depth-First

Pick first element of Q; Add path extensions to front of Q

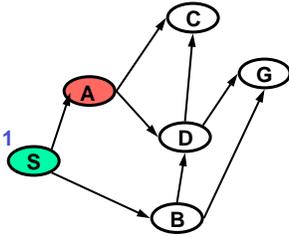
	Q
1	(S)
2	
3	
4	
5	



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S)
3	
4	
5	

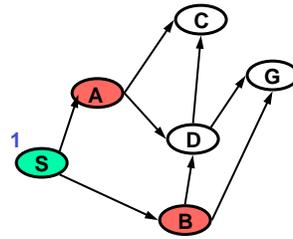


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	
4	
5	



Added paths in blue

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Simple Search Algorithm

Let Q be a list of partial paths,
 Let S be the start node and
 Let G be the Goal node.

1. Initialize Q with partial path <S>
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
 - a) Remove N from Q
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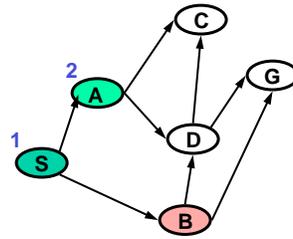
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Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(AS) (BS)
3	
4	
5	

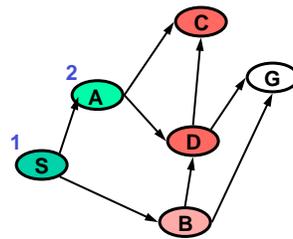


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(AS) (BS)
3	(CAS) (DAS) (BS)
4	
5	

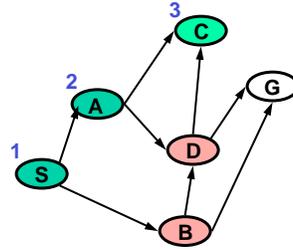


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	
5	

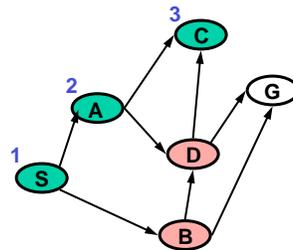


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	

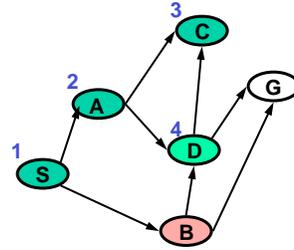


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	

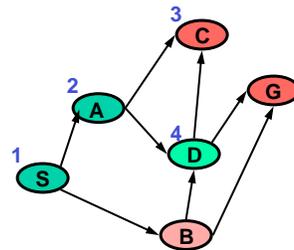


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

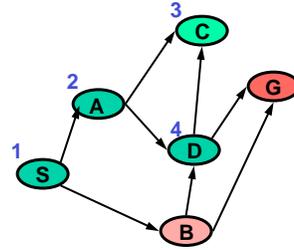
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

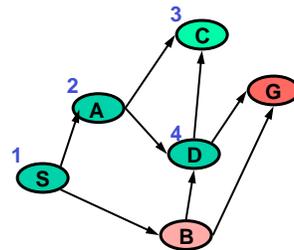
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S) (G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

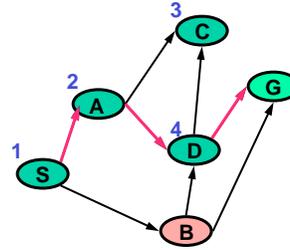
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S) (G D A S) (B S)
6	(G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S) (G D A S) (B S)
6	(G D A S) (B S)



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Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. Initialize Q with partial path <S>
2. If Q is empty, fail. Else, pick a partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else:
 - a) Remove N from Q
 - b) Find all children of head(N) and create a one-step extension of N to each child
 - c) Add all extended paths to Q
 - d) Go to step 2

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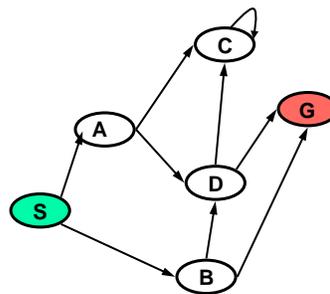
Problem Solving as State Space Search

- Problem Formulation (Modeling)
- Formal Representation
- Reasoning Algorithms
 - A generic search algorithm description
 - Depth-first search example
 - Handling cycles
 - Breadth-first search example

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Issue: Starting at S and moving top to bottom, will depth-first search ever reach G?



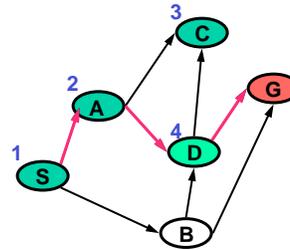
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Depth-First

Effort can be wasted in more mild cases

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S) (D A S) (B S)
6	(G D A S) (B S)



- C visited multiple times
- Multiple paths to C, D & G

How much wasted effort can be incurred in the worst case?

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How Do We Avoid Repeat Visits?

Idea:

- Keep **track** of **nodes** already **visited**.
- **Do not** place expanded path on **Q** if head is a **visited node**.

Does this maintain correctness?

- Any **goal reachable** from a **node** that was **visited a second** time would be **reachable** from that node the **first time**.

Does this always improve efficiency?

- **Visits** only a **subset** of the **original paths**, such that each node appears at most once at the head of a visited path.

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How Do We Modify The Simple Search Algorithm?

Let Q be a list of partial paths,
Let S be the Start node and
Let G be the Goal node.

1. Initialize Q with partial path <S> as only entry;
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
 - a) Remove N from Q
 - b) Find all children of head(N) and create a one-step extension of N to each child
 - c) Add to Q all the extended paths
 - d) Go to step 2

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Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. Initialize Q with partial path <S> as only entry; set Visited = {}
2. If Q is empty, fail. Else, pick some partial path N from Q
3. If head(N) = G, return N (goal reached!)
4. Else
 - a) Remove N from Q
 - b) Find all children of head(N) not in Visited and create a one-step extension of N to each child
 - c) Add to Q all the extended paths
 - d) Add children of head(N) to Visited
 - e) Go to step 2

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Testing for the Goal

- This algorithm stops (in step 3) when $\text{head}(N) = G$.
- We could have performed this test in step 6 as each extended path is added to Q. This would catch termination earlier and be perfectly correct for all the searches we have covered so far.
- However, performing the test in step 6 will be incorrect for the optimal search algorithms that we look at later. We have chosen to leave the test in step 3 to maintain uniformity with these future searches.

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Problem Solving as State Space Search

- Problem Formulation (modeling)
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 - Handling cycles
 - Breadth-first search example

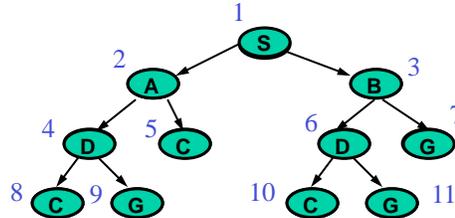
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Breadth First Search (BFS)

Idea: After visiting node

- Visit its **children left to right**
- Visit its **siblings**, before its **children**



Assume we remove the **first element** of Q,

Where to Q do we **add** the path extensions?

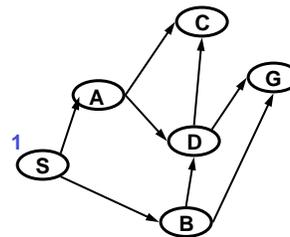
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2		
3		
4		
5		
6		



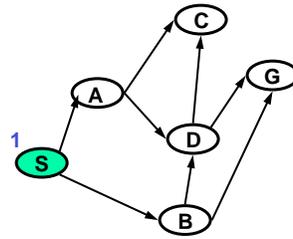
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

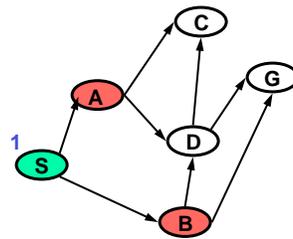
	Q	Visited
1	(S)	S
2		
3		
4		
5		
6		



Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

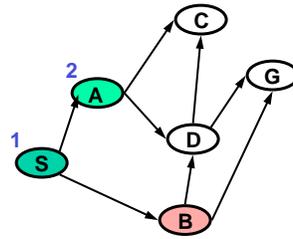
	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3		
4		
5		
6		



Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3		
4		
5		
6		



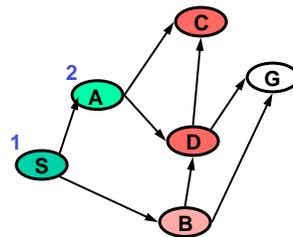
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



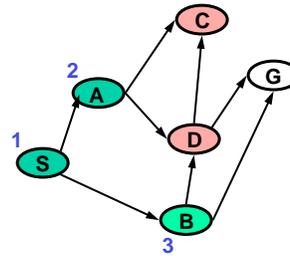
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



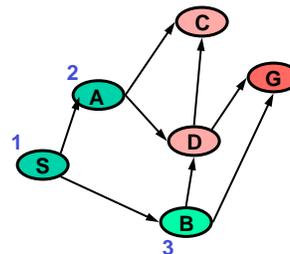
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5		
6		



* We could stop here, when the first path to the goal is generated.

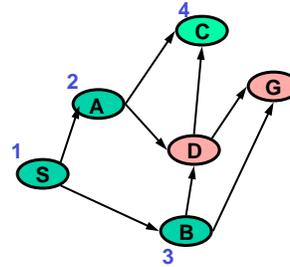
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5		
6		



* We could stop here, when the first path to the goal is generated.

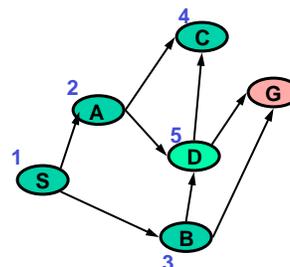
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S
6		



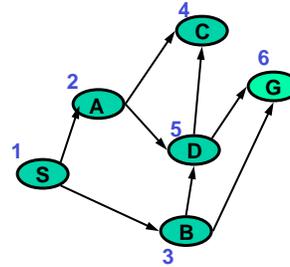
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S
6	(G B S)	G,C,D,B,A,S



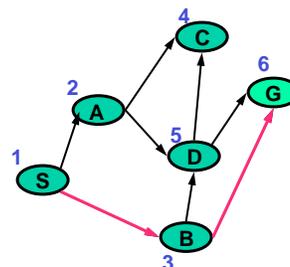
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Breadth-First with Visited List

Pick first element of Q; Add path extensions to end of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S
6	(G B S)	G,C,D,B,A,S



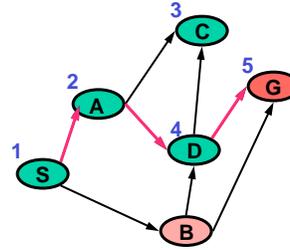
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Depth-first with Visited List

Pick first element of Q; Add path extensions to front of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A, B, S
3	(C A S) (D A S) (B S)	C, D, B, A, S
4	(D A S) (B S)	C, D, B, A, S
5	(G D A S) (B S)	G, C, D, B, A, S

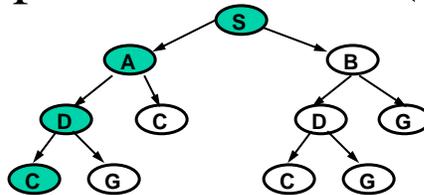


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For each search type, where do we place the children on the queue?

Depth First Search (DFS)

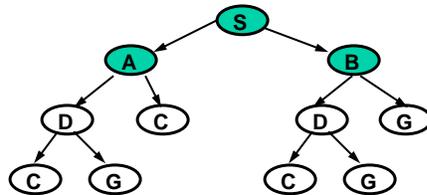


Depth-first:

Add path extensions to **front** of Q

Pick first element of Q

Breadth First Search (BFS)



Breadth-first:

Add path extensions to **back** of Q

Pick first element of Q

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What You Should Know

- Most problem solving tasks may be formulated as **state space search**.
- State space search is **formalized** using **graphs, simple paths, search trees**, and pseudo code.
- **Depth-first** and **breadth-first search** are framed, among others, as instances of a **generic search strategy**.
- **Cycle detection** is required to achieve efficiency and completeness.

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Appendix

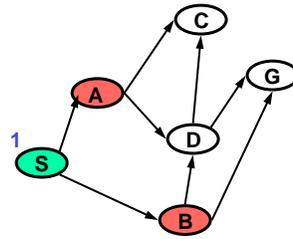
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Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	
3	
4	
5	
6	
7	



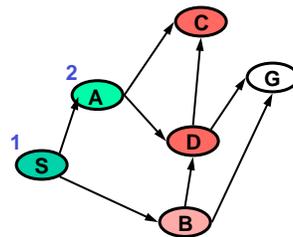
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Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	
4	
5	
6	
7	



Added paths in blue

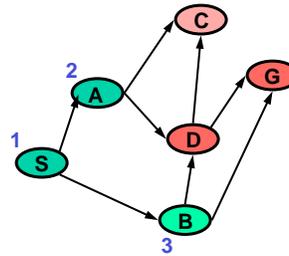
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100

Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	
5	
6	
7	



Added paths in blue

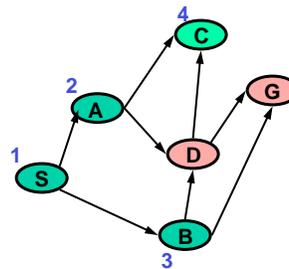
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101

Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	
6	
7	



Added paths in blue

Revisited nodes in pink

* We could have stopped here, when the first path to the goal was generated.

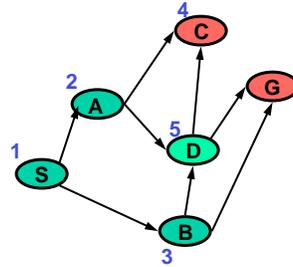
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Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	(D A S) (D B S) (G B S)
6	
7	



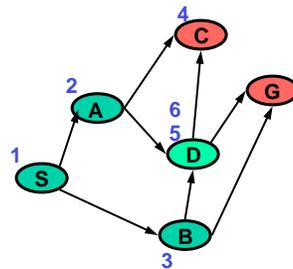
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103

Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	(D A S) (D B S) (G B S)
6	(D B S) (G B S) (C D A S) (G D A S)
7	



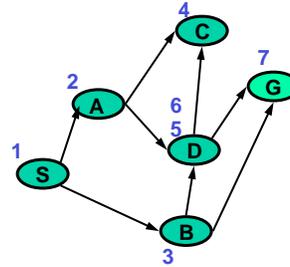
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Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	(D A S) (D B S) (G B S)
6	(D B S) (G B S) (C D A S) (G D A S)
7	(G B S) (C D A S) (G D A S) (C D B S) (G D B S)



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