

16.410/13 — Problem Set 10

Problem 1: Markov Chains (50 points)

Part A: Markov Chain Representation

A die is rolled repeatedly. Which of the following can be represented as a Markov Chain? For those that can be, provide the transition matrix

- The largest number X_n shown up to the n th roll.
- The number N_n of sixes in n rolls.
- At time r , the time C_r till the most recent six.
- At time r , the time B_r until the next six.

Part B: Absent-minded TA (30 points)

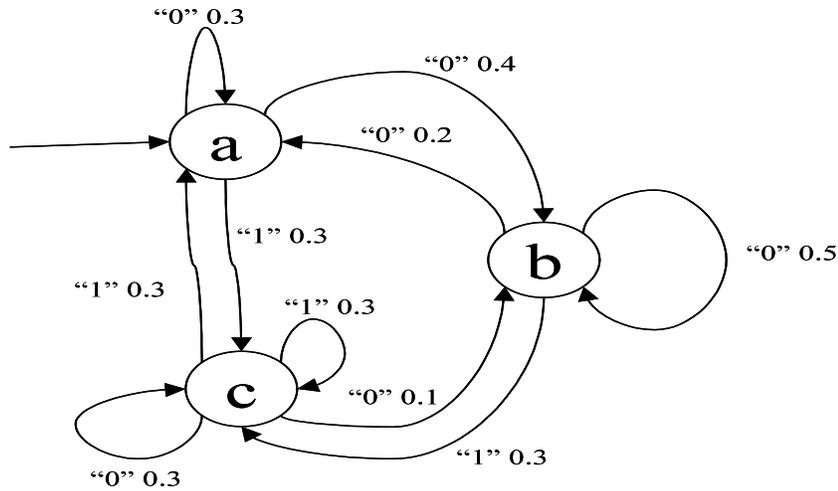
An absent-minded TA has two umbrellas. He commutes between his home and office every day. If it is rainy during that commute, he takes an umbrella with him to his destination; if there is no umbrella at his current location, he gets wet... If it is not raining, he does not take any umbrellas with him during his commute.

You are interested in computing the frequency that this TA gets wet. Assume that the probability that it rains during a given commute is p independently from every other commute.

- Represent this problem as a Markov chain. Show the transition graph and write down the transition matrix. (*HINT: The state should be the number of umbrellas at TAs current location*)
- Assume that initially the TA starts at his home, where he has two umbrellas. And assume that $p = 1/3$ (which incidentally happens to be the frequency of rainy days in Boston on November). Compute the probability that he gets wet on the 5th day. Compute the same probability for the 10th, 100th, 150th, 200th, and 250th days.
- Repeat the exercise described in the previous item when the TA starts at home, but both his umbrellas are in his office.

Problem 2: HMMs: Most Likely State Sequence (50 points)

A biologist is studying a very simple marine life form that can make two types of limb movements. These limb movements are labeled 0 and 1. The limb movements cause the organism to move in the water and find new sources of food. The very simple control mechanism of the organism has been modeled by the biologist as a Hidden Markov Model (HMM), consisting of three states - **a**, **b**, and **c** - with probabilistic transitions as shown in the diagram below.



While the organism's state is not directly observable, limb movement sequences ($\mathbf{m}_{1,t}$) can be observed and recorded from video tapes of the organism. The organism is always in state **a** when it has finished feeding, so that when it begins to move to a new location it is known to be in state **a**. After one such feeding period the following sequence of limb movements is recorded "00101101". Use the Viterbi algorithm to estimate the most likely sequence of states that the organism moved through for the given limb movement sequence. Show your work by completing the table below.

t	1	2	3	4	5	6	7	8	9
	0	0	1	0	1	1	0	1	
$\gamma_a(t)$									
$\gamma_b(t)$									
$\gamma_c(t)$									

What is the most likely sequence of states for the given sequence of limb movements?

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