16.410/13 — Problem Set 9

Problem 1: Mixed Integer Linear Programming (50 points)

Part A: 3-SAT (10 points)

In this problem we will prove that solving a Mixed Integer Linear Program is NP-hard. Consider the (in)famous 3-SAT problem. This problem is to find an assignment to the binary variables $\{x_{ij} | i \in 1 \ldots N, j \in \{1, 2, 3\}\}$ that satisfies

$$\bigwedge_i \bigvee_j x_{ij}$$

Formulate this problem as a MILP. 3-SAT is the first NP-complete problem. Being able to solve this MILP in polynomial time would imply that P=NP.

Part B: Map Coloring (10 points)

Suppose that you are a map maker and it is time to fill in your, N beautiful countries. There is a rule that no two adjacent countries can be the same color, however, a country and it's colonies (or other disjoint units) must be the same color. You can make N distinct colors but each one costs a different amount to produce. Formulate a MILP that would tell you how to color the map most cheaply.

Part C: Knapsack

Suppose that you are a burgler with a knapsack of finite weight capacity c. The house into which you have broken has N items of value v_i and weight w_i for $i \in 1 \ldots N$. You are trying to steal greatest value from a house, but can only carry what will fit in your knapsack. The items themselves are not divisible.

Part C.1 (10 points)

Formulate and solve the relaxed problem in which the items are divisible. There is no need to break out simplex. The solution is intuitive—you can reason it out directly.

Part C.2 (20 points)

By hand work out the solution using Branch and Bound with the following weights and values.

$$w = [2.15, 2.75, 3.35, 3.55, 4.20, 5.80]^T$$

$$v = [1, 1.5, 2, 2.5, 3.5, 5]^T$$

$$c = 15.05$$

Problem 2: Probabilistic Reasoning (50 points)

Part A: Fundamentals of Probability (30 points)

Recall from the lecture that the axioms of probability are:

- For any event A, $\mathtt{P}(A) \geq 0$.
- $\mathtt{P}(U) = 1$.
- If $A \cap B = \emptyset$, then $\mathtt{P}(A \cup B) = \mathtt{P}(A) + \mathtt{P}(B)$.

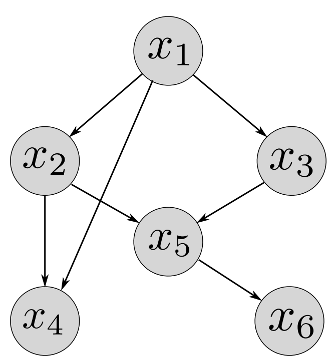
From these axioms, prove the following two statements:

- **Continuity of probability measures:** If A and B are two events such that $A \subseteq B$, then $\mathtt{P}(A) \leq \mathtt{P}(B)$.
- **Boole's Inequality (a.k.a. union bound):** If $\{A_1, A_2, \ldots, A_n\}$ is a finite collection of events (not necessarily disjoint), then

$$\mathtt{P}\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mathtt{P}(A_i).$$

Part B: Bayesian Networks (20 points)

Consider the following Bayes Net. Write out an expression for the joint probability distribution of the variables involved. Assume that each variable can take two different values. Comment on the complexity of inference (that is, the size of the tables that you will need to store) to compute the marginal probability distribution of x_6 from the conditional probabilities. Repeat the same exercise for x_4.



16.410 / 16.413 Principles of Autonomy and Decision Making
Fall 2010