16.36 Communication Systems Engineering
Spring 2009

Lectures 6: Modulation

Eytan Modiano

Modulation

- Digital signals must be transmitted as analog waveforms
- Baseband signals
  - Signals whose frequency components are concentrated around zero
- Passband signals
  - Signals whose frequency components are centered at some frequency f_c away from zero
- Baseband signals can be converted to passband signals through modulation
  - Multiplication by a sinusoid with frequency f_c

Baseband signals

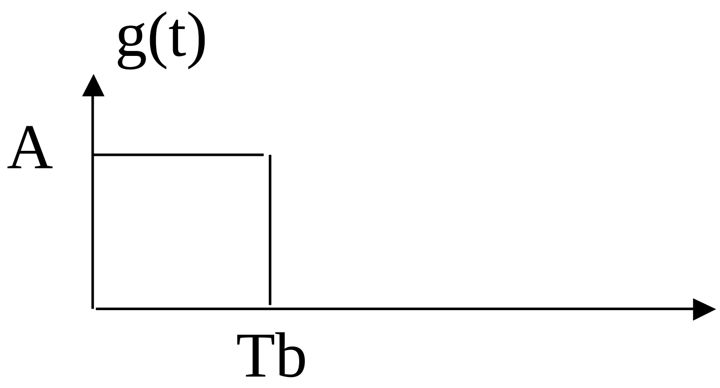
- **The simplest signaling scheme is pulse amplitude modulation (PAM)**
  - **With binary PAM a pulse of amplitude A is used to represent a "1" and a pulse with amplitude -A to represent a "0"**

- **The simplest pulse is a rectangular pulse, but in practice other type of pulses are used**
  - **For our discussion we will usually assume a rectangular pulse**

- **If we let g(t) be the basic pulse shape, than with PAM we transmit g(t) to represent a "1" and -g(t) to represent a "0"**

$$1 \Rightarrow S(t) = g(t)$$

$$0 \Rightarrow S(t) = -g(t)$$



M-ary PAM

- **Use M signal levels, $A_1 \ldots A_M$**
  - **Each level can be used to represent $\text{Log}_2(M)$ bits**

- **E.g., $M = 4 \Longrightarrow A_1 = -3, A_2 = -1, A_3 = 1, A_4 = 3$**
  - **$S_i(t) = A_i\, g(t)$**

- **Mapping of bits to signals: each signal can be used to represent $\text{Log}_2(M)$ bits**
  - **Does the choice of bits matter? Yes - more on Gray coding later**

S_i	$b_1 b_2$
S_1	00
S_2	01
S_3	11
S_4	10

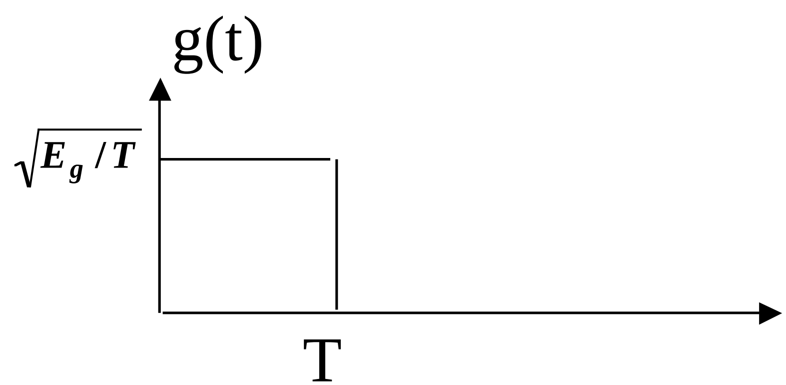
Signal Energy

$$E_m = \int_0^T (S_m(t))^2 dt = (A_m)^2 \int_0^T (g_t)^2 dt = (A_m)^2 E_g$$

- **The signal energy depends on the amplitude**
- **E_g is the energy of the signal pulse g(t)**
- **For rectangular pulse with energy E_g =>**

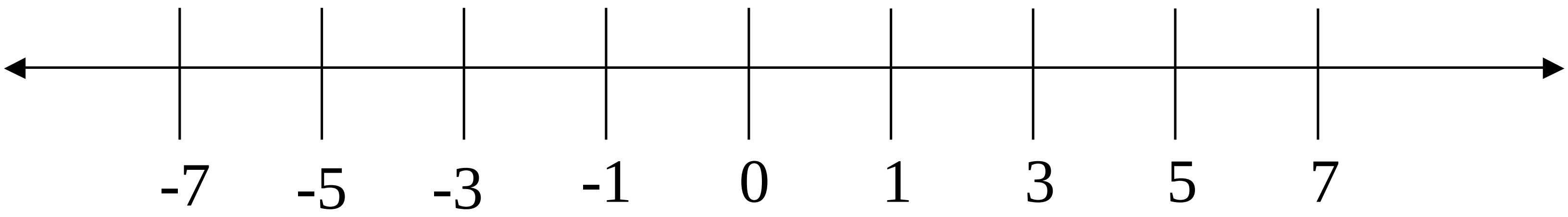
$$E_g = \int_0^T A^2 dt = TA^2 \Rightarrow A = \sqrt{E_g / T}$$

$$g(t) = \begin{cases} \sqrt{E_g / T} & 0 \le t \le T \\ 0 & otherwise \end{cases}$$



Symmetric PAM

- **Signal amplitudes are equally distant and symmetric about zero**



A_m = (2m-1-M), m=1…M

E.g., M = 4 $\Rightarrow$ A_1 = -3, A_2 = -1, A_3 = 1, A_4 = 3

- **Average energy per symbol:**

$$E_{ave} = \frac{E_g}{M}\sum_{m=1}^{M}(2m-1-M)^2 = E_g(M^2-1)/3$$

- **What about average energy per bit?**

Gray Coding

- **Mechanism for mapping bits to symbols so that the number of bit errors is minimized**
  - **Most likely symbol errors are between adjacent levels**
  - **Want to MAP bits to symbols so that the number of bits that differ between adjacent levels is minimized**

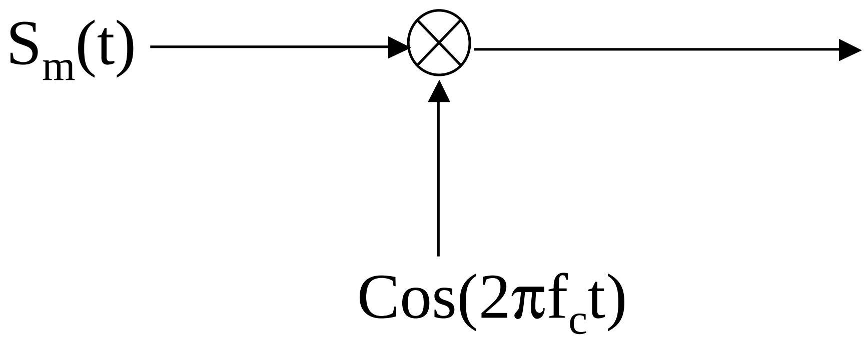
- **Gray coding achieves 1 bit difference between adjacent levels**

- **Example M= 8 (can be generalized to any M which is a power of 2)**
  - **Also see the case of M = 4 from earlier slide**

A_1	000
A_2	001
A_3	011
A_4	010
A_5	110
A_6	111
A_7	101
A_8	100

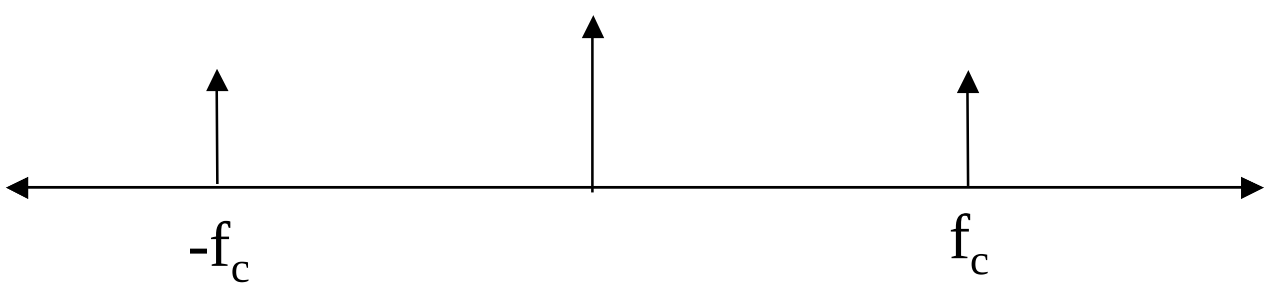
Bandpass signals

- **To transmit a baseband signal S(t) through a pass-band channel at some center frequency f_c, we multiply S(t) by a sinusoid with that frequency**



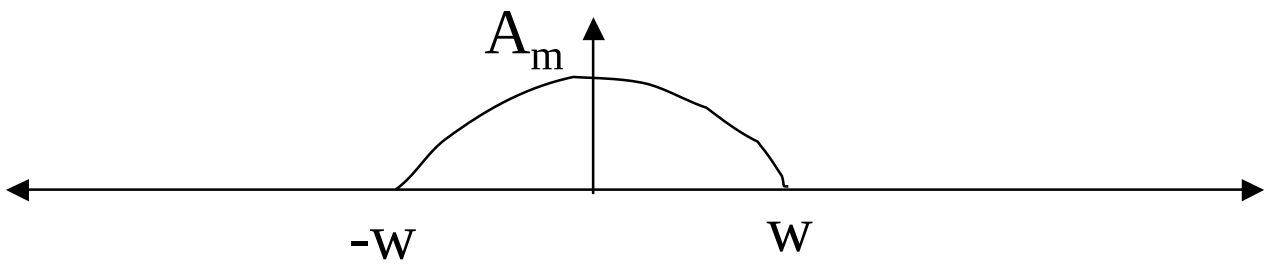
$$U_m(t) = S_m(t)Cos(2\pi f_c t)$$
$$= A_m g(t)\ Cos(2\pi f_c t)$$

$$F[Cos(2\pi f_c t)] = (\delta(f-f_c)+\delta(f+f_c))/2$$

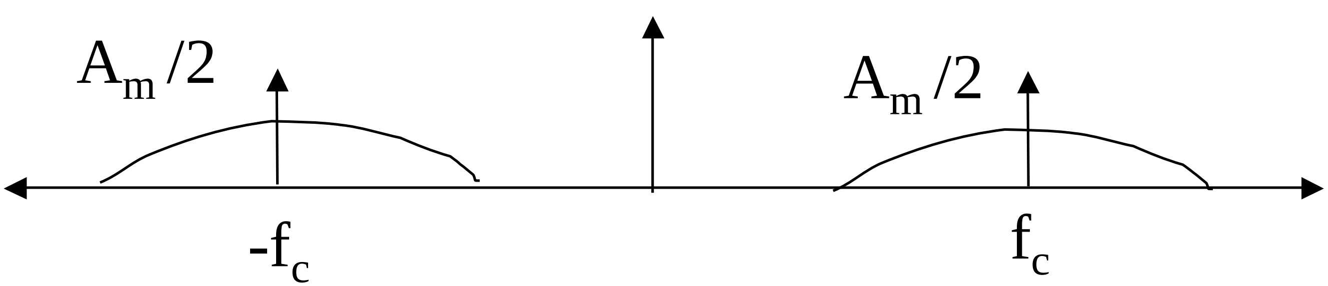


Bandpass signals, cont.

$F[A_m g(t)]$ = depends on g()



$F[A_m g(t)\ Cos(2\pi f_c t)]$



Recall: Multiplication in time = convolution in frequency

Energy content of modulated signals

$$E_m = \int_{-\infty}^{\infty} U_m^2(t)dt = \int_{-\infty}^{\infty} A_m^2 g^2(t) Cos^2(2\pi f_c t)dt$$

$$Cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$$

$$E_m = \frac{A_m^2}{2}\int_{-\infty}^{\infty} g^2(t) + \underbrace{\frac{A_m^2}{2}\int_{-\infty}^{\infty} g^2(t) Cos(4\pi f_c t)dt}_{\approx 0}$$

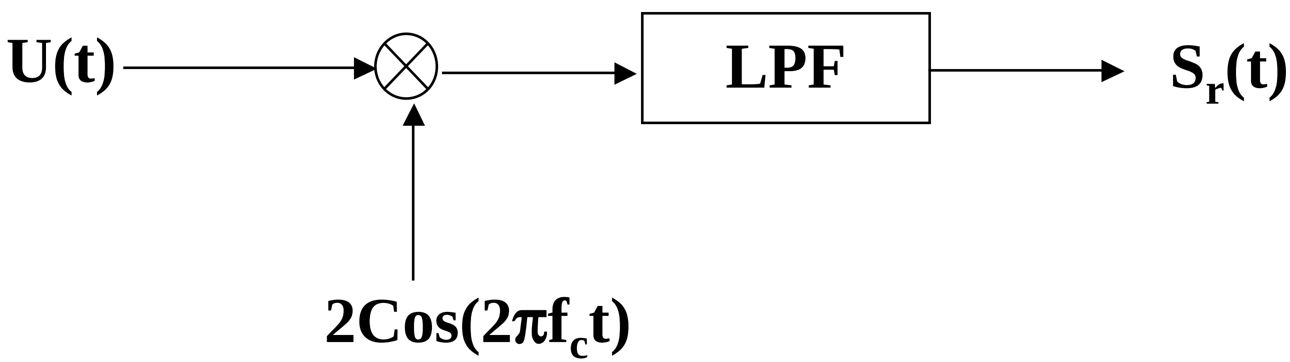
$$E_m = \frac{A_m^2}{2} E_g$$

- **The cosine part is fast varying and integrates to 0**
- **Modulated signal has 1/2 the energy as the baseband signal**

Demodulation

- To recover the original signal, multiply the received signal ($U_m(t)$) by a cosine at the same frequency

$$U_m(t) = S_m(t)Cos(2\pi f_c t) = A_m g(t)\ Cos(2\pi f_c t)$$

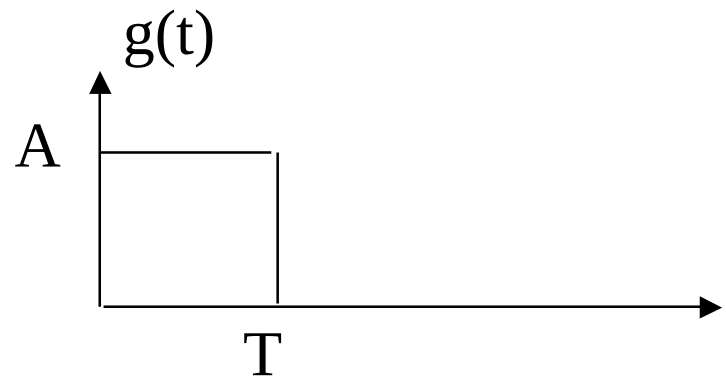


$$U(t)2Cos(2\pi f_c t) = 2S(t)Cos^2(2\pi f_c t) = S(t) + S(t)Cos(4\pi f_c t)$$

- The high frequency component is rejected by the LPF and we are left with S(t)

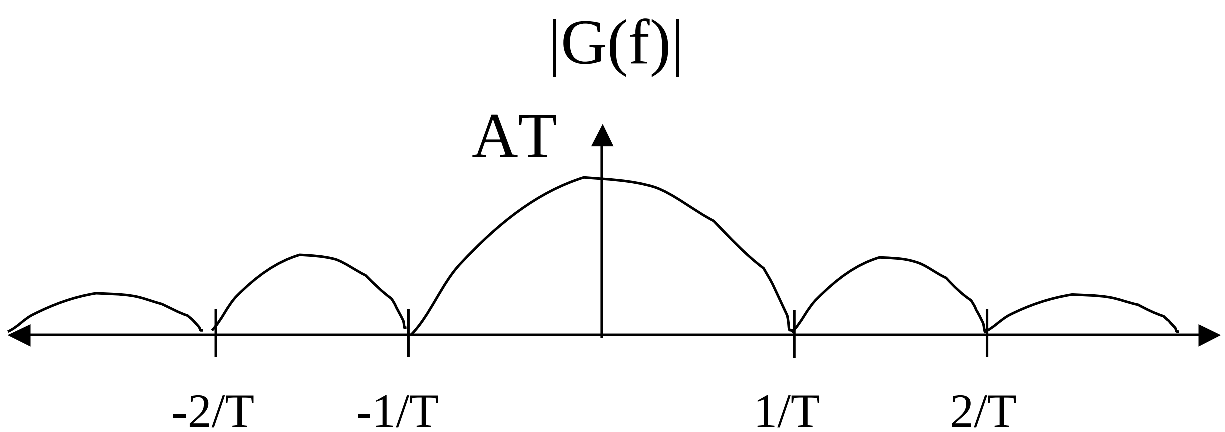
Bandwidth occupancy
(ideal rectangular pulse)

$$G(f) = F[g(t)]$$



$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = \int_{0}^{T} A e^{-j2\pi ft} dt$$

$$G(f) = (AT) Sinc(\pi f T) e^{-j\pi f T}$$



- **Ideal rectangular pulse has unlimited bandwidth**
  - **First "null" bandwidth = 2(1/T) = 2/T**
- **In practice, we "shape" the pulse so that most of its energy is contained within a small bandwidth**

Bandwidth efficiency

- R_s = symbol rate = 1/T
  - $Log_2(M)$ bits per symbol => R_b = bit rate = $\log_2(M)/T$ bits per second
- BW = 2/T = $2R_s$
  - Bandwidth efficiency = R_b/BW = $\log_2(M)/T * (T/2) = \log_2(M)/2$ BPS/Hz
- Example:
  - M = 2 $\Rightarrow$ bandwidth efficiency = 1/2
  - M = 4 $\Rightarrow$ bandwidth efficiency = 1
  - M = 8 => bandwidth efficiency = 3/2
- Increased BW efficiency with increasing M
- However, as M increase we are more prone to errors as symbols are closer together (for a given energy level)
  - Need to increase symbol energy level in order to overcome errors
  - Tradeoff between BW efficiency and energy efficiency

Energy utilization

$$E_{ave} = \frac{E_g}{M}\sum_{m=1}^{M}(2m-1-M)^2 = E_g(M^2-1)/3,\ E_g = \text{basic pulse energy}$$

After modulation $E_u = \frac{E_s}{2} = E_g(M^2-1)/6$

$$E_b = \text{average energy per bit} = \frac{(M^2-1)}{6Log_2(M)}E_g$$

- **Average energy per bit increases as M increases**