

MIT OpenCourseWare
<http://ocw.mit.edu>

16.346 Astrodynamics
Fall 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Exercises 22

Space Navigation from Earth to Mars

A space vehicle is en route to Mars, and one-tenth year after departure from earth the vector positions in astronomical units of the vehicle and the two planets are as follows:

$$\mathbf{r} = \begin{bmatrix} 0.1041415 \\ 0.9982173 \\ 0.0185441 \end{bmatrix} \quad \mathbf{r}_E = \begin{bmatrix} 0.1552486 \\ 0.9721359 \\ 0 \end{bmatrix} \quad \mathbf{r}_M = \begin{bmatrix} -1.0769734 \\ 1.2512651 \\ 0.0531597 \end{bmatrix}$$

in a heliocentric, ecliptic oriented coordinate system.

1. A position fix is made by measuring the following angles:

1. Angle between Mars and Sirius
2. Angle between Mars and Beta Centauri
3. Angle between earth and Beta Centauri

with the lines of sight to the stars Sirius and Beta Centauri given by

$$\mathbf{i}_{\text{Sirius}} = \begin{bmatrix} -0.1800747 \\ 0.7493110 \\ -0.6372645 \end{bmatrix} \quad \mathbf{i}_{\text{Beta Centauri}} = \begin{bmatrix} -0.4300127 \\ -0.5750170 \\ -0.6960205 \end{bmatrix}$$

Determine the rms uncertainty in position in statute miles assuming that the standard deviation of the measurement errors is 0.05 milliradian. For simplicity, assume that the measurements are all made simultaneously and at precisely the reference time.

2. If the additional measurement of the

4. Angle between the sun and Mars

is made, determine the reduction in the rms uncertainty in position.

Note: Assume that all four measurements have equal weight.

Note: Use the recursive form of the estimator to incorporate the fourth measurement.

Answer: With three measurements the error should be approximately 16,000 miles and with four measurements, approximately 9,000 miles.

3. Let \mathbf{i}_{*1} and \mathbf{i}_{*2} be two unit vectors in the direction of two selected stars. Two measurements are made on board the spacecraft of the angles A_1 and A_2 between the lines of sight to a planet and the two stars. If \mathbf{i}_r is the unit vector describing the direction of the spacecraft from the planet, then we have

$$\mathbf{i}_r \cdot \mathbf{i}_{*1} = -\cos A_1$$

$$\mathbf{i}_r \cdot \mathbf{i}_{*2} = -\cos A_2$$

- a) Derive the equations for The Line of Position given on Page 1 of Lecture 21.
- b) Use the data from the Problem 1 first to calculate A_1 , the angle between Mars and Sirius; then calculate A_2 , the angle between Mars and Beta Centauri. Finally, determine \mathbf{i}_r , the direction from Mars to the spacecraft, from the expressions derived above.