16.346 Astrodynamics
Fall 2008

Exercises 06

The equations describing the hyperbolic locus of velocity vectors are not valid when the transfer angle is $\theta = 180°$. However, by expressing the velocity vector components in polar coordinates rather than along skewed-axes, the singularity disappears.

1. Show that the velocity components along skewed axes are related to ordinary polar coordinate components as

$$v_c = v_{\theta_1} \csc \phi_1 = \frac{c}{r_2} v_{\theta_1} \csc \theta$$

$$v_\rho = v_{r_1} - v_{\theta_1} \cot \phi_1 = v_{r_1} - \frac{r_2 \cos \theta - r_1}{r_2 \sin \theta} v_{\theta_1}$$

Hint: Equate the velocity vector in both sets of coordinates and calculate the scalar product with each of the unit vectors.

2. Derive the equation of the hyperbolic locus of velocity vectors in the form

$$v_{r_1} v_{\theta_1} \sin \theta - v_{\theta_1}^2 \left(\cos \theta - \frac{r_1}{r_2} \right) = \frac{\mu}{r_1} (1 - \cos \theta)$$

and show that it exhibits no difficulties as θ approaches 180 degrees.

Refer to Figure 6.2