16.346 Astrodynamics
Fall 2008

Exercises 02

1. The angular momentum and eccentricity vectors of an orbit are

$$\mathbf{h} = 2\sqrt{\frac{\mu}{3}}\,\mathbf{i}_z \qquad \mathbf{e} = -\tfrac{1}{3}(2\,\mathbf{i}_x + \mathbf{i}_y)$$

Find the position and velocity vectors $\mathbf{r}$ and $\mathbf{v}$ when the direction of the position vector is $\mathbf{i}_r = \mathbf{i}_x$. (Use $\mu = 4\pi^2$)

Answer: $\mathbf{r} = 4\,\mathbf{i}_x$ and $\mathbf{v} = \dfrac{\pi}{\sqrt{3}}(\mathbf{i}_x + \mathbf{i}_y)$

2. **Prob. 4–8** To derive the polar equation of an ellipse with the origin of coordinates at the center of the ellipse (See Lecture 2, Page 3), we may consider the triangle CPF where r is the radius from the center C to a point P on the ellipse and F is the focus of the ellipse.

The sides of the triangle are

$$PF = a - ex = a - er\cos\theta \qquad CF = ae \qquad CP = r$$

We can use the Law of Cosines for the triangle

$$(a - ex)^2 = (a - er\cos\theta)^2 = r^2 + a^2e^2 - 2aer\cos\theta$$

which gives $\quad r^2(1 - e^2\cos^2\theta) = a^2(1 - e^2) = b^2 \quad$ or

$$\boxed{r = \frac{b}{\sqrt{1 - e^2\cos^2\theta}}}$$

$$\begin{aligned} 1\,\text{mile} &= 1.609347221\,\text{km} \\ 1\,\text{au} &= 149,597,870.00\,\text{km} \\ 1\,\text{au} &= 92,955,620.79\,\text{miles} \\ 1\,\text{au/day} &= 1078.822025\,\text{miles/sec} \\ 1\,\text{au/day} &= 5,696,180.29\,\text{feet/sec} \end{aligned}$$