16.333 Lecture # 8

Aircraft Lateral Dynamics

Spiral, Roll, and Dutch Roll Modes

Aircraft Lateral Dynamics

- Using a procedure similar to the longitudinal case, we can develop the equations of motion for the **lateral dynamics**

$$\dot{x} = Ax + Bu \; , \; x = \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} \; , \; u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

and $\dot{\psi} = r \sec\theta_0$

$$A = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \frac{Y_r}{m} - U_0 & g\cos\theta_0 \\ (\frac{L_v}{I'_{xx}} + I'_{zx}N_v) & (\frac{L_p}{I'_{xx}} + I'_{zx}N_p) & (\frac{L_r}{I'_{xx}} + I'_{zx}N_r) & 0 \\ (I'_{zx}L_v + \frac{N_v}{I'_{zz}}) & (I'_{zx}L_p + \frac{N_p}{I'_{zz}}) & (I'_{zx}L_r + \frac{N_r}{I'_{zz}}) & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}$$

where

$$\begin{aligned} I'_{xx} &= (I_{xx}I_{zz} - I_{zx}^2)/I_{zz} \\ I'_{zz} &= (I_{xx}I_{zz} - I_{zx}^2)/I_{xx} \\ I'_{zx} &= I_{zx}/(I_{xx}I_{zz} - I_{zx}^2) \end{aligned}$$

and

$$B = \begin{bmatrix} (m)^{-1} & 0 & 0 \\ 0 & (I'_{xx})^{-1} & I'_{zx} \\ 0 & I'_{zx} & (I'_{zz})^{-1} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{bmatrix}$$

Lateral Stability Derivatives

- A key to understanding the lateral dynamics is **roll-yaw coupling**.
- L_p rolling moment due to roll rate:
  - Roll rate p causes right to move wing down, left wing to move up
    $\longrightarrow$ Vertical velocity distribution over the wing $W = py$
  - Leads to a spanwise change in the AOA: $\alpha_r(y) = py/U_0$
  - Creates lift distribution (chordwise strips)

$$\delta L_w(y) = \frac{1}{2}\rho U_0^2 C_{l_\alpha} \alpha_r(y) c_y dy$$

  - Net result is higher lift on right, lower on left
  - Rolling moment:

$$L = \int_{-b/2}^{b/2} \delta L_w(y) \cdot (-y) dy = -\frac{1}{2}\rho U_0^2 \int_{-b/2}^{b/2} C_{l_\alpha} \frac{py^2}{U_0} c_y dy \Rightarrow L_p < 0$$

  - **Key point: positive roll rate $\Rightarrow$ negative roll moment.**

- L_r rolling moment due to yaw rate:
  - Positive r has left wing advancing, right wing retreating
    $\longrightarrow$ Horizontal velocity distribution over wing $U = U_0 - ry$
  - Creates lift distribution over wing (chordwise strips)

$$L_w(y) \sim \frac{1}{2}\rho U^2 C_l c dy \approx \frac{1}{2}\rho (U_0^2 - 2U_0 ry) C_l c_y dy$$

  - Net result is higher lift on the left, lower on the right.
  - Rolling Moment: $L = \int_{-b/2}^{b/2} L_w(y) \cdot (-y) dy \approx \rho U_0 r \int_{-b/2}^{b/2} C_l c_y y^2 dy$
  - For large aspect ratio rectangular wing (crude)

$$L_r \approx (\frac{1}{6} \text{ to } \frac{1}{4}) C_L > 0$$

  - **Key point: positive yaw rate $\Rightarrow$ positive roll moment.**

- N_p yawing moment due to roll rate:
  - Rolling wing induces a change in spanwise AOA, which changes the spanwise **lift** and **drag**.
  - Distributed drag change creates a yawing moment. Expect higher drag on right (lower on left) $\rightarrow$ positive yaw moment
  - There is both a change in the lift (larger on downward wing because of the increase in α) and a rotation (leans forward on downward wing because of the larger α). $\rightarrow$ negative yaw moment
  - In general hard to know which effect is larger. Nelson suggests that for a rectangular wing, crude estimate is that

$$N_p \approx \frac{1}{2}\rho U_0^2 Sb(-\frac{C_L}{8}) < 0$$

- N_r yawing moment due to yaw rate:
  - Key in determining stability properties – mostly from fin.
  - Positive r has fin moving to the left which increases the apparent angle of attack by

$$\Delta\alpha_f = \frac{rl_f}{(U_0)_f}$$

  - Creates increase in lift at the tail fin by

$$\Delta L_f = \frac{1}{2}\rho(U_0^2)_f S_f C_{L_{\alpha_f}} \Delta\alpha_f$$

  - Creates a change in the yaw moment of

$$N = -l_f \Delta L_f = -\frac{1}{2}\rho(U_0)_f S_f C_{L_{\alpha_f}} r l_f^2$$

  - So $N_r = -\frac{1}{2}\rho(U_0)_f S_f C_{L_{\alpha_f}} l_f^2 < 0$
  - **Key point: positive yaw rate $\Rightarrow$ negative yaw moment.**

	L	N
p	< 0	?
r	> 0	< 0

Numerical Results

- The code gives the numerical values for all of the stability derivatives. Can solve for the eigenvalues of the matrix A to find the modes of the system.

$$\begin{aligned} &-0.0331 \pm 0.9470i \\ &-0.5633 \\ &-0.0073 \end{aligned}$$

  – Stable, but there is one very slow pole.

- There are 3 modes, but they are a **lot more complicated** than the longitudinal case.

Slow mode	-0.0073	$\Rightarrow$	**Spiral Mode**
Fast real	-0.5633	$\Rightarrow$	Roll Damping
Oscillatory	$-0.0331 \pm 0.9470i$	$\Rightarrow$	**Dutch Roll**

Can look at normalized eigenvectors:

	Spiral	Roll	Dutch Roll	
$\beta = w/U_0$	0.0067	-0.0197	0.3269	-28°
$\hat{p} = p/(2U_0/b)$	-0.0009	-0.0712	0.1198	92°
$\hat{r} = r/(2U_0/b)$	0.0052	0.0040	0.0368	-112°
ϕ	1.0000	1.0000	1.0000	0°

Not as enlightening as the longitudinal case.

Lateral Modes

Roll Damping - well damped.

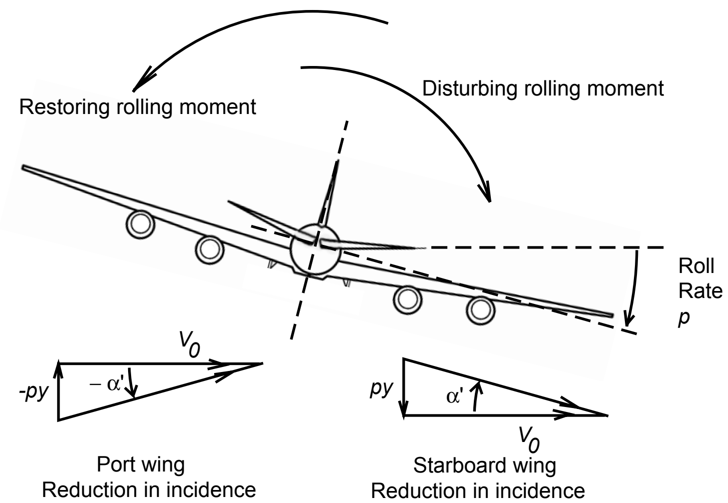
– As the plane rolls, the wing going down has an increased α (wind is effectively "coming up" more at the wing)

– Opposite effect for other wing.

– There is a difference in the lift generated by both wings $\longrightarrow$ more on side going down

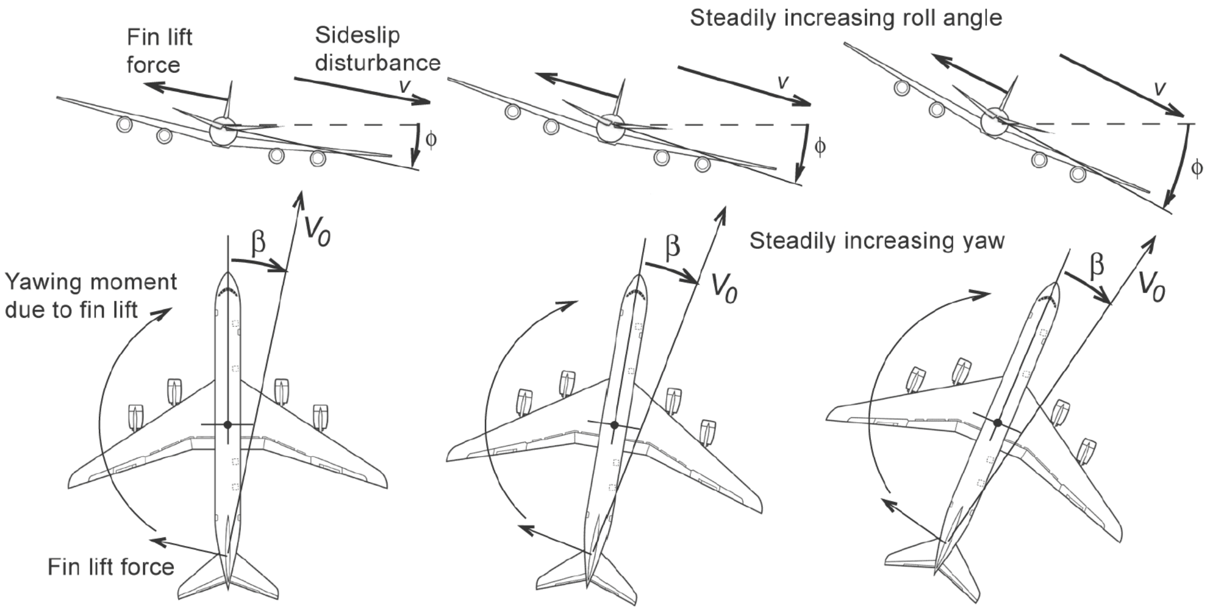
– The differential lift creates a **moment** that tends to **restore** the equilibrium. Recall that $L_p < 0$

– After a disturbance, the roll rate builds up exponentially until the restoring moment balances the disturbing moment, and a steady roll is established.



Spiral Mode - slow, often unstable.

- From level flight, consider a disturbance that creates a small roll angle $\phi > 0 \rightarrow$ This results in a small side-slip v (vehicle *slides downhill*)
- Now the tail fin hits on the oncoming air at an incidence angle β $\rightarrow$ extra tail lift $\rightarrow$ positive yawing moment
- Moment creates positive yaw rate that creates positive roll moment ($L_r > 0$) that increases the roll angle and tends to increase the side-slip
  $\rightarrow$ makes things worse.
- If unstable and left unchecked, the aircraft would fly a slowly diverging path in roll, yaw, and altitude $\Rightarrow$ it would tend to *spiral* into the ground!!



- Can get a restoring torque from the wing **dihedral**

- Want a small tail to reduce the impact of the spiral mode.

Dutch Roll - damped oscillation in yaw, that couples into roll.

- Frequency similar to longitudinal short period mode, not as well damped (fin less effective than horizontal tail).
- Consider a disturbance from straight-level flight

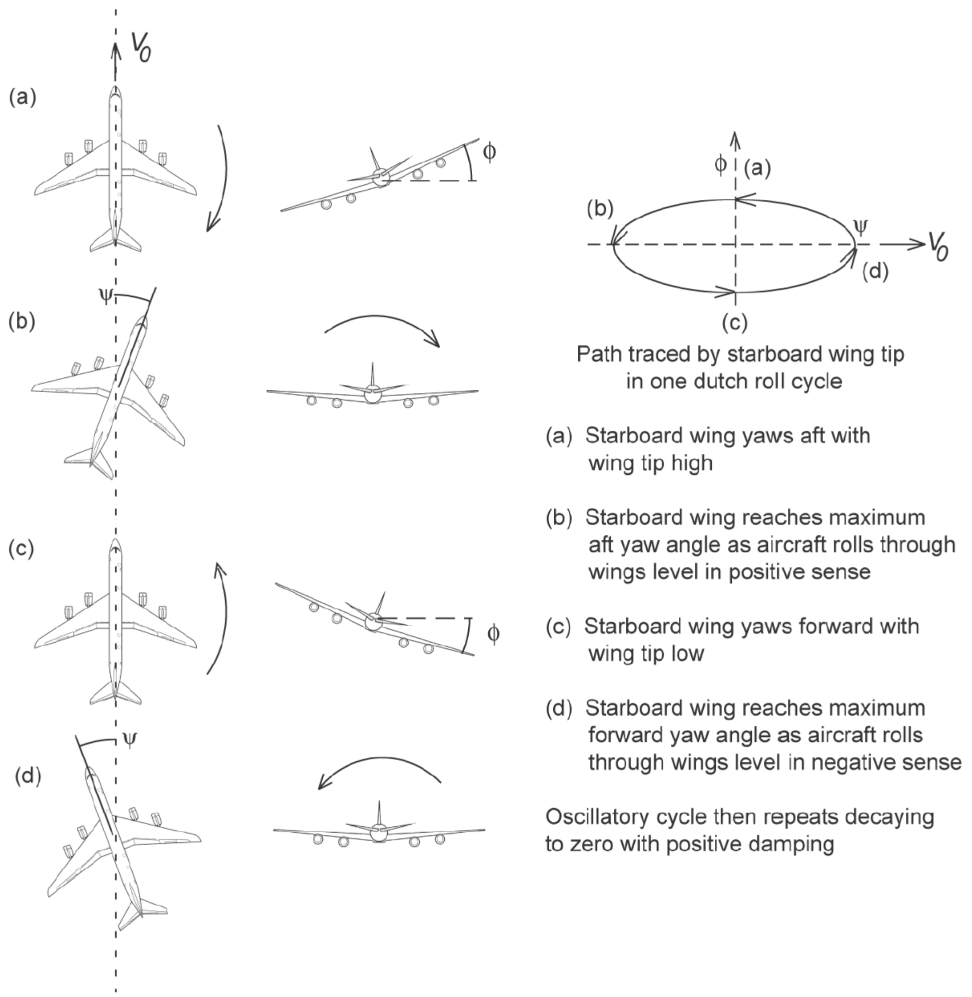
  → Oscillation in yaw ψ (fin provides the *aerodynamic stiffness*)

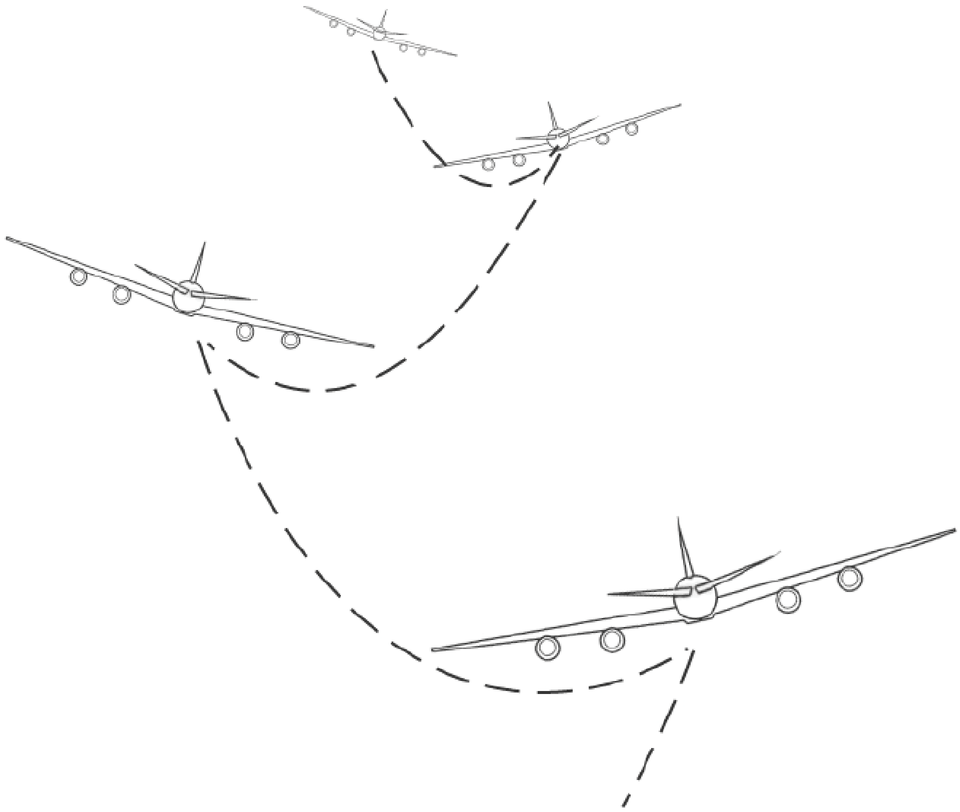
  → Wings moving back and forth due to yaw motion result in oscillatory differential lift/drag (wing moving forward generates more lift) $L_r > 0$

  → Oscillation in roll ϕ that lags ψ by approximately 90°

  ⇒ *Forward going wing is low*

  Oscillating roll ⇒ sideslip in direction of low wing.





- Do you know the origins on the name of the mode?
- Damp the Dutch roll mode with a large tail fin.

Aircraft Actuator Influence

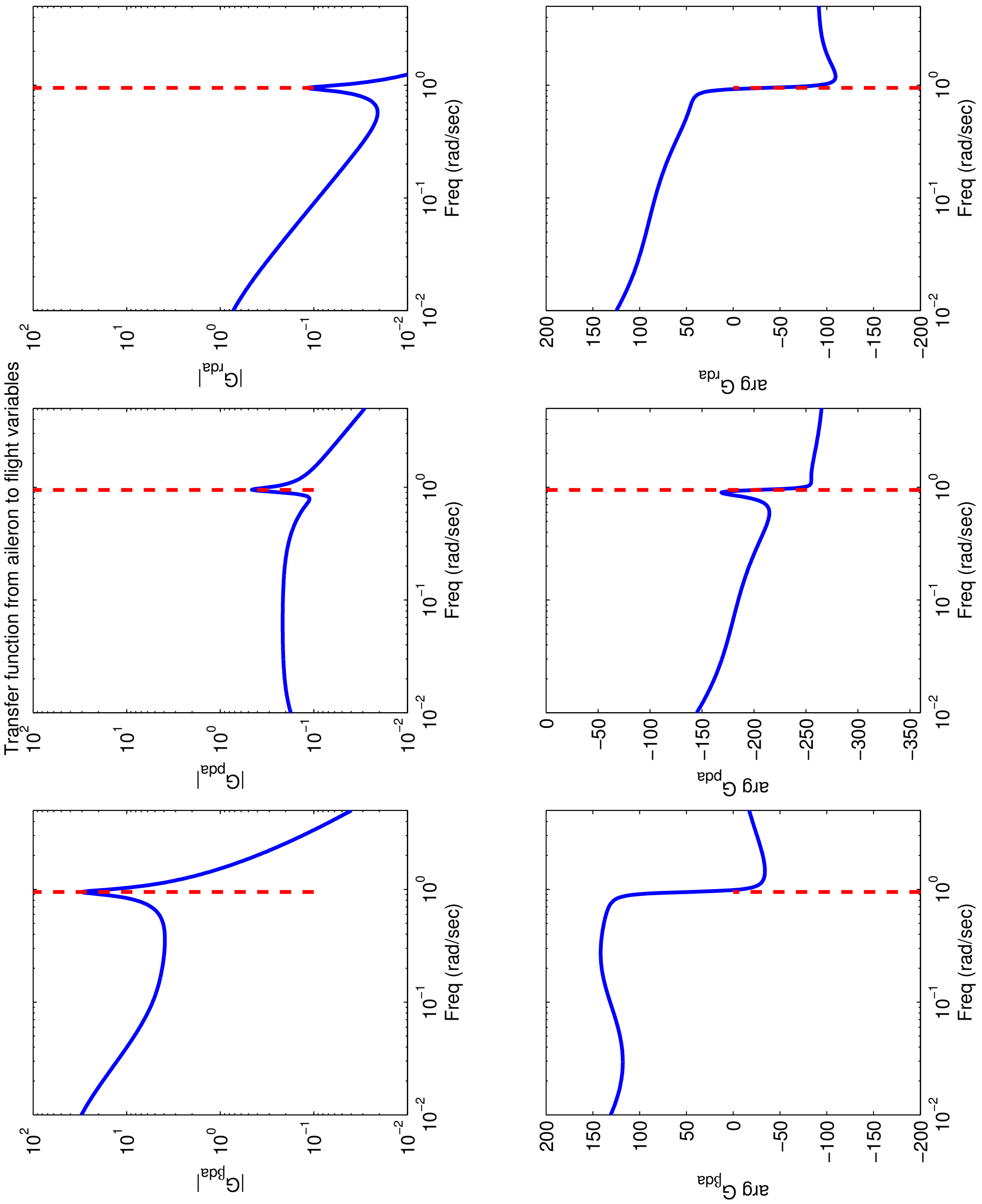


Figure 1: Aileron impulse to flight variables. Response primarily in ϕ.

- Transfer functions dominated by lightly damped Dutch-roll mode.
- Note the rudder is physically quite high, so it also influences the A/C roll.
- Ailerons influence the Yaw because of the differential drag

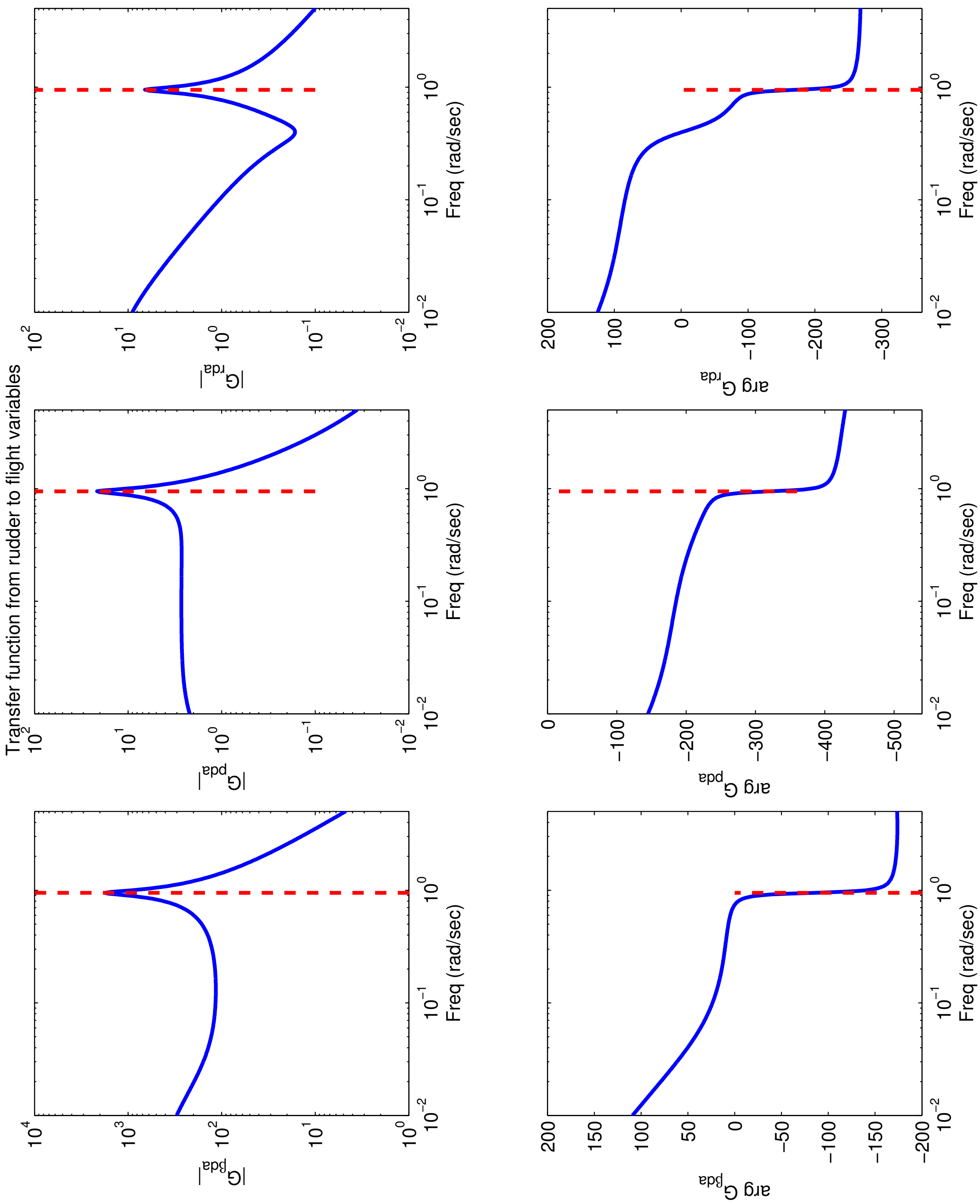


Figure 2: Aileron impulse to flight variables. Response primarily in ϕ.

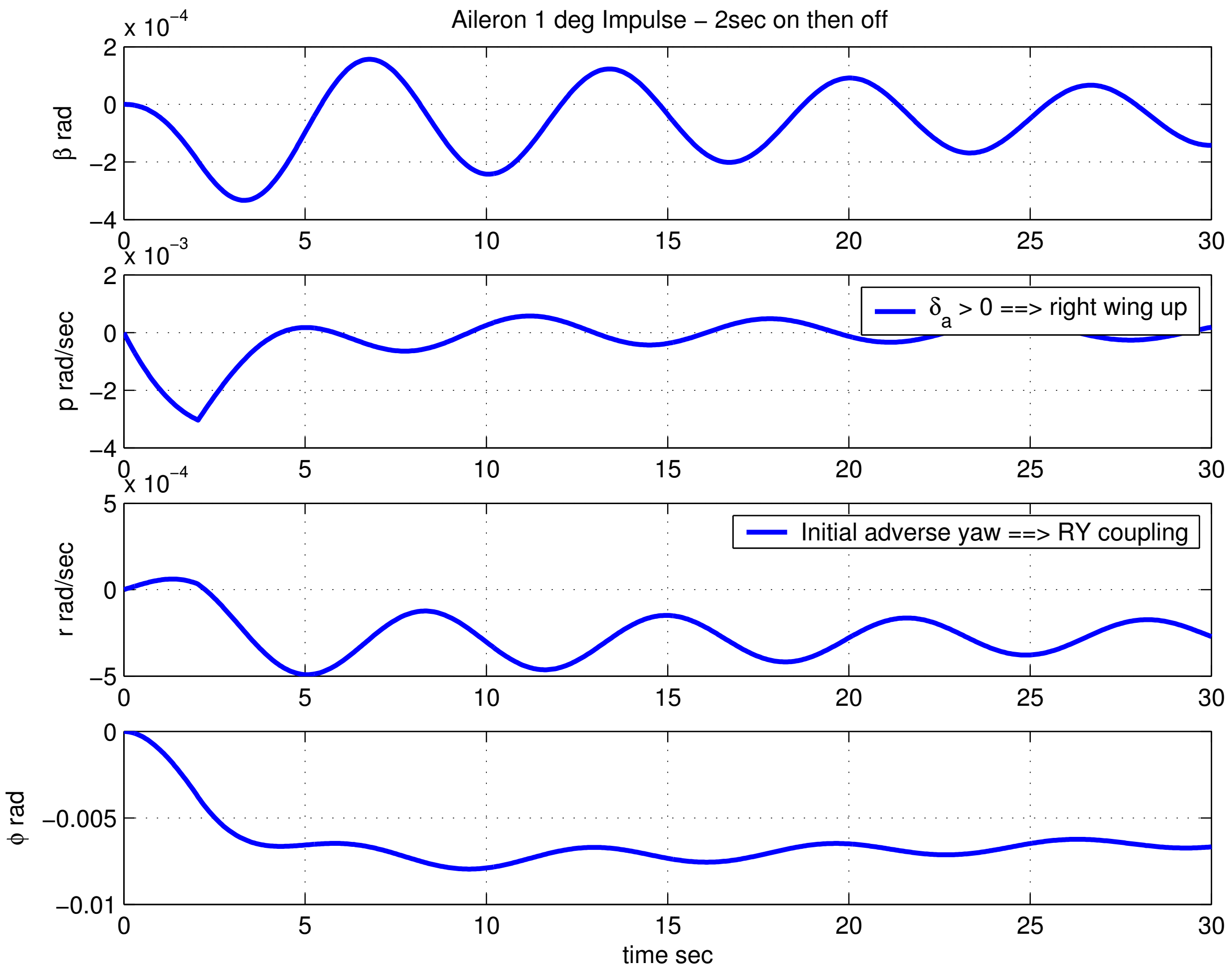


Figure 3: Aileron impulse to flight variables

- **Aileron** δ_a =1deg impulse for 2 sec.
  - Since $\delta_a > 0$ then right aileron goes down, and right wing goes up $\rightarrow$ Reid's notation, and it is **not** consistent with the picture on 6–4 (from Nelson).
  - Influence of the roll mode seen in the response of p to application and release of the aileron input.
  - See effect of *adverse yaw* in the yaw rate response caused by the differential drag due to aileron deflection.
  - Spiral mode harder to see.
  - Dutch mode response in other variables clear (1 rad/sec $\sim$ 6 sec period).

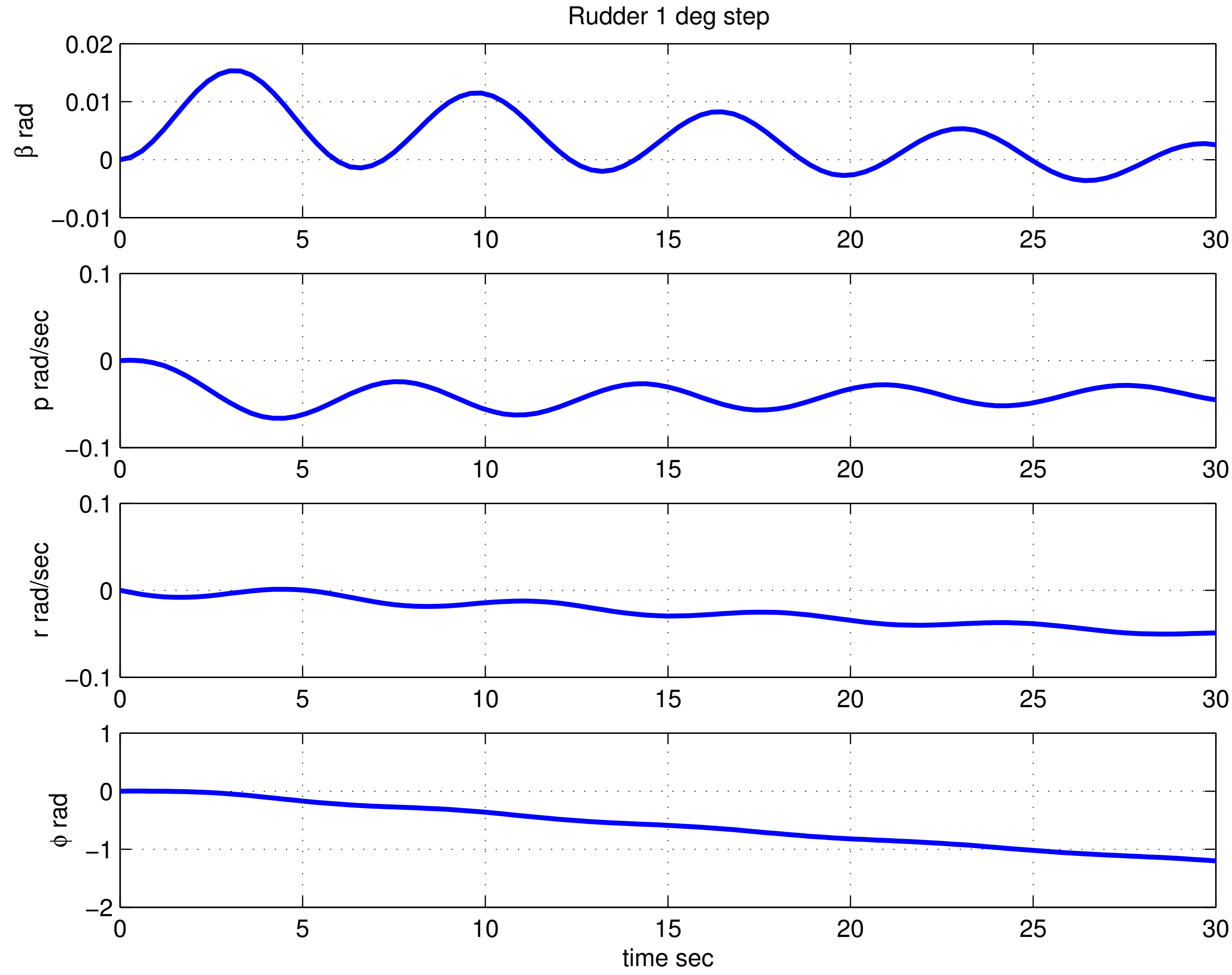


Figure 4: Rudder step to flight variables

- **Rudder step input** 1deg step.
  - Dutch roll response very clear. Other 2 modes are much less pronounced.
  - β shows a very lightly damped decay.
  - p clearly excited as well. Doesn't show it, but often see evidence of adverse roll in p response where initial p is opposite sign to steady state value. Reason is that the forces act on the fin which is well above the *cg* $\rightarrow$ and the aircraft responds rapidly (initially) in roll.
  - ϕ ultimately oscillates around 2.5°