16.333: Lecture #15

Inertial Sensors

Complementary filtering

Simple Kalman filtering

INERTIAL SENSORS

- MOST, IF NOT ALL, INERTIAL SENSORS MEASURE RATES (LINEAR OR ANGULAR) OR ACCELERATIONS
  - WHILE THE RATE INFORMATION IS TYPICALLY VALID OVER LONG PERIODS OF TIME, IT MUST BE INTEGRATED TO GET DISPLACEMENTS

⇒ EVEN VERY SMALL ERRORS IN THE RATES CAUSE UNBOUNDED GROWTH IN THE INTEGRATED QUANTITIES.

- THUS NEED TO CAREFULLY CHARACTERIZE THE ERROR MODEL FOR YOUR SENSOR
  - AND PROVIDE PERIODIC RESETS VIA AN EXTERNAL SENSOR.

- NOTE: INERTIAL SENSORS TYPICALLY MEASURE RATES IN THE BODY FRAME. MUST MAP THESE TO FIXED FRAME THROUGH THE EULER ANGLES.
  - INTRODUCES MORE ERROR.

- CAN RELATE THE BODY FRAME ATTITUDE WRT EARTH (INERTIAL?) USING EULER ANGLES OR QUATERNIONS

  ⇒ BODY FRAME RATES MAP TO DERIVATIVES OF THE EULER ANGLES

  ⇒ 3 NL DIFFERENTIAL EQUATIONS FOR ϕ, θ, ψ

  ⇒ CAN SOLVE NUMERICALLY FOR VEHICLE ATTITUDE

  ⇒ USE THESE ANGLES TO TRANSFORM MEASURED ACCELERATIONS/VELOCITIES INTO INERTIAL FRAME ⇒ $\dot{x}, \dot{y}, \dot{z}$ EQUATIONS

- ERRORS IN THE ATTITUDE ESTIMATES WILL COUPLE INTO THE POSITION ERRORS

  ⇒ NEED A "REFERENCE" - LOW FREQ BUT VERY ACCURATE (NO DRIFT)

  ⇓

  GPS PERFECT FOR THIS

GYROS

- SPINNING ROTOR CONSTRAINED TO FOLLOW THE ROTATION THE GYRO EXPERIENCES ABOUT ITS INPUT AXIS

- TORQUE REQUIRED TO CONSTRAIN ROTOR PROPORTIONAL TO INPUT RATE

$$T = H\dot{\theta}$$

$$\hookrightarrow J\omega_R$$

- PRECESSION EFFECT
- GYRO ACCURACY DETERMINED BY PARASITIC TORQUES (MASS SHIFTS)

  ⇒ LOT OF EFFORT EXERTED ON FIXING THIS PROBLEM

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- IN THE CASE $\theta = 90°$, THE ROTOR SPIN AXIS IS IN THE HORIZONTAL PLANE

(Image removed due to copyright considerations.)

$$\Rightarrow M_Y = I_s \dot{\phi} \dot{\psi}$$

$$\dot{\phi} = \frac{M_Y}{I_s \dot{\psi}}$$

- NOW EXPLICIT THAT IF WE APPLY A MOMENT TO A GYROSCOPE ABOUT AN AXIS PERPENDICULAR TO ITS AXIS OF SPIN, THE GYROSCOPE WILL PRECESS ABOUT AN AXIS PERPENDICULAR TO BOTH THE SPIN AXIS AND THE MOMENT AXIS.

$$\left.\begin{array}{l} -\ \text{TORQUE ABOUT } \vec{2}_Y \text{ AXIS} \\ -\ \text{SPIN ABOUT } \vec{2}_Z \text{ AXIS} \end{array}\right\} \text{PRECESS ABOUT } \vec{2}_X \text{ (VERTICAL)}$$

- DIRECTION OF PRECESSION:

CAUSES POSITIVE END OF SPIN AXIS TO ROTATE TOWARDS POSITIVE END OF MOMENT AXIS.

TUNING FORK GYROS - MEMS

- EXPLOITS CORIOLIS EFFECT WHEN A VIBRATING MASS SUBJECTED TO A RATE OF ROTATION ABOUT AN AXIS // PLANE OF VIBRATION

$$\vec{a} \simeq \vec{v} \times \vec{\Omega}$$

  ⇒ DRIVE VIBRATIONS IN ONE DIRECTION

  ⇒ ROTATION OF BASE RESULTS IN ADDITIONAL VIBRATIONS

- CAN BE VERY SMALL
- TEMP DEPENDENT → CALIBRATE

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OPTICAL GYROS

- SEND LIGHT AROUND A ROTATING PATH + MEASURE THE CW - CCW TIMES

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⇒ SAGNAC EFFECT

$$\Delta T = \frac{4A}{c^2}\dot{\theta}$$

- **RING LASER GYRO**
  - MEASURE FRINGE PATTERN BY INTERFERING CW/CCW BEAMS
  - INPUT ROTATION CAUSES PATTERN TO MOVE AT A RATE $\sim \dot{\theta}$

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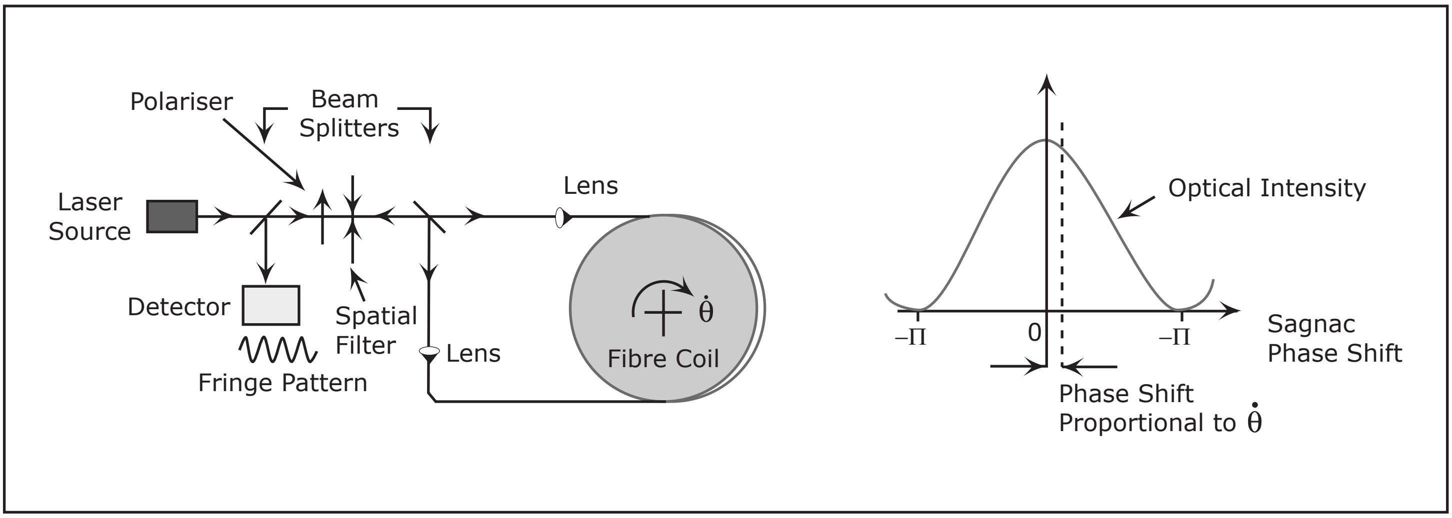
  - VERY ACCURATE
  - INSENSITIVE TO ACCEL
  - STABLE SCALE FACTOR
  - VERY EXPENSIVE

INTERFEROMETRIC FIBRE OPTIC GYRO (IFOG)

- ALSO USES SAGNAC EFFECT
- COIL OF N-TURN FIBRE



- PHASE SHIFT OF CW/CCW RESULTS IN REDUCTION OF LIGHT INTENSITY AT DECTECTOR

$$\Phi_s = \frac{2\pi L D}{\lambda_c}\dot{\theta}$$

- AVOIDS SOME OF THE PRECISE MACHINING OF RLG
- CAN USE CHEAPER ELECTRONICS
- SMALLER THAN RLG

$\Rightarrow$ CHEAPER, BUT STILL EXPENSIVE

- BIGGEST PROBLEM WITH GYROS ARE THE BIAS AND BIAS DRIFT.
  - BIAS CAUSED BY:
    - DRIVE EXCITATION FEEDTHROUGH
    - OUTPUT ELECTRONICS OFFSETS
    - BEARING TORQUES.
  - 3 TYPES:
    - FIXED BIAS
    - BIAS VARIATION FROM ONE TURN-ON TO ANOTHER. (THERMAL) [BIAS STABILITY]
    - BIAS DRIFT, USUALLY MODELED AS A RANDOM WALK.

$$\delta\omega_{BIAS} = \delta\omega_{CONST} + \delta\omega_{BS} + \delta\omega_{BD}$$

WITH $\frac{d}{dt}\delta\omega_{BD} = w(t)$; $w \sim N(0, Q)$

Q - KNOWN, WITH UNITS OF $(\text{DEG}/h)\sqrt{h}$

- TYPICAL #'S

	GOOD	MED	BAD
BIAS STABILITY DEG/SEC	10^{-7}	10^{-5}	10^{-2}
BIAS DRIFT $(\text{DEG}/\sqrt{\text{SEC}})$	10^{-5}	10^{-4}	10^{-3}

NOISE MODELS

- SYSTRON DONNER QUARTZ GYRO

  ⇒ "DOES NOT HAVE A STABLE BIAS"

  FTN OF THE INPUT RATE!

- TYPICAL MODEL: ANGULAR VELOCITY ω GIVEN BY

$$\omega = \omega_m - b - n_r$$

BIAS DRIFT

$$\dot{b} = n_\omega$$

$$E[n_\omega] = 0$$

$$E\left[n_\omega(t) n_\omega^T(t')\right] = N_\omega \delta(t-t')$$

DRIFT-RATE NOISE

~ GAUSSIAN

$$E[n_r] = 0$$

$$E\left[n_r(t) n_r'(t')\right] = N_r \delta(t-t')$$

AND $E\left[n_w(t) n_r'(t')\right] = 0 \quad \forall t, t'$

TYPICAL RESULTS: $\sigma_r = \sqrt{N_r} = 0.009 (°/\text{sec})/\sqrt{\text{Hz}}$

$\sigma_w = \sqrt{N_w} = 0.00050 \; (°/\text{sec})\sqrt{\text{Hz}}$

- MUST INCLUDE THE DRIFT IN THE BIAS IN THE KALMAN FILTER DYNAMICS

TYPICAL PROBLEMS

- SCALE FACTOR NONLINEARITY
  - INPUT KNOWN RATE ω
  - MEASURE ω_g
  - PLOT $\epsilon = \omega_g - \omega$

E-CORE RD2100 FOG

(Image removed due to copyright considerations.)

- CHANGE TEMPERATURE OF GYRO AND MEASURE ERROR → $\epsilon \simeq b_0(\omega) + b_1(\omega)T + b_2(\omega)T^2$

(Image removed due to copyright considerations.)

- CAN CALIBRATE, BUT THIS IS CLEARLY A BIT OF A PROBLEM.

Examples of Estimation Filters from Recent Aircraft Projects at MIT

November 2004

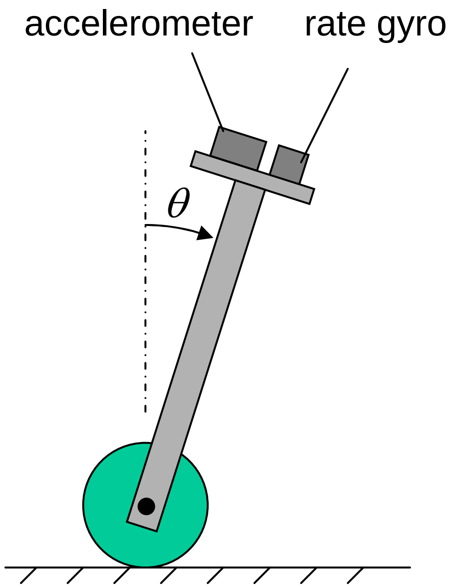
Sanghyuk Park and Jonathan How

Complementary Filter (CF)

Often, there are cases where you have *two* different measurement sources for estimating *one* variable and the noise properties of the two measurements are such that one source gives good information only in low frequency region while the other is good only in high frequency region.

→ You can use a complementary filter !

Example : Tilt angle estimation using accelerometer and rate gyro

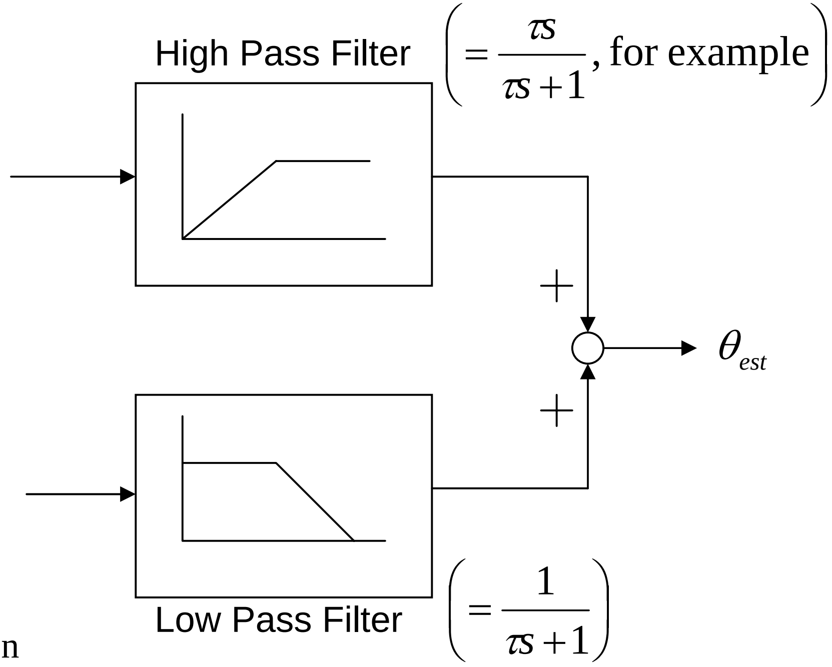


$$\theta \approx \int (\text{angular rate})\, dt$$

- not good in long term due to integration

$$\theta \approx \sin^{-1}\left(\frac{\text{accel. output}}{g}\right)$$

- only good in long term
- not proper during fast motion



$$\left(=\frac{\tau s}{\tau s+1}, \text{ for example}\right)$$

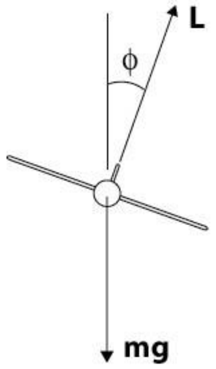
$$\left(=\frac{1}{\tau s+1}\right)$$

Complementary Filter(CF) Examples

- CF1. Roll Angle Estimation
- CF2. Pitch Angle Estimation
- CF3. Altitude Estimation
- CF4. Altitude Rate Estimation

CF1. Roll Angle Estimation

- High freq. : integrating roll rate (p) gyro output
- Low freq. : using aircraft kinematics
  - Assuming steady state turn dynamics,
    roll angle is related with turning rate, which is close to yaw rate (r)

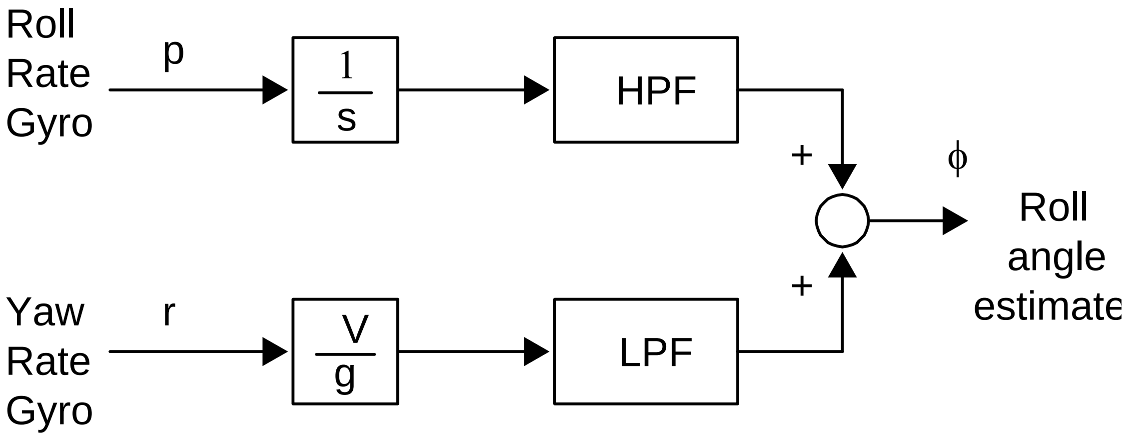


$$L\sin\phi = mV\Omega$$
$$L \approx mg$$
$$\Omega \approx r$$
$$\sin\phi \approx \phi$$

⇨

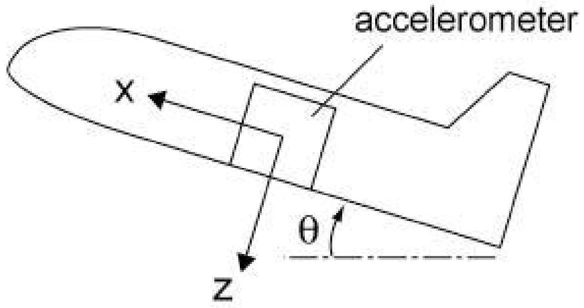
$$\phi \approx \frac{V}{g} r$$

<u>CF setup</u>



CF2. Pitch Angle Estimation

- High freq. : integrating pitch rate (q) gyro output
- Low freq. : using the sensitivity of accelerometers to gravity direction
  - "*gravity aiding*"



In steady state

$$A_X = g\sin\theta \qquad A_Z = -g\cos\theta \quad \Rightarrow \quad \theta = \tan^{-1}\left(-\frac{A_x}{A_z}\right)$$

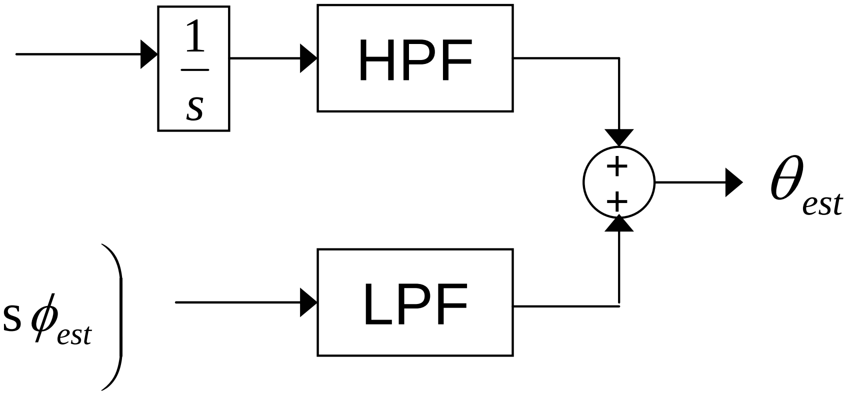
A_X , A_Z – accelerometer outputs

- Roll angle compensation is needed

CF setup

q_{meas}, ϕ_{est} $\Rightarrow$ $\dot{\theta} \approx q_{meas}\cos\phi_{est}$

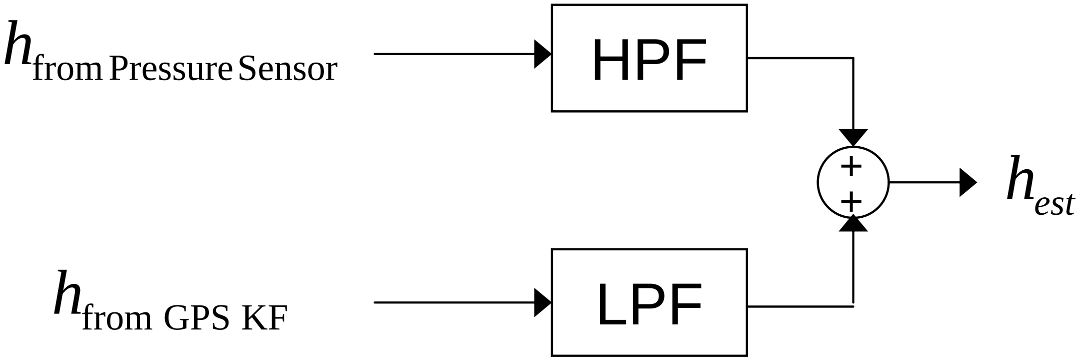
A_x, A_z, ϕ_{est} $\Rightarrow$ $\theta_{ss} = \tan^{-1}\left(-\frac{A_x}{A_z}\cos\phi_{est}\right)$



CF3. Altitude Estimation

- Motivation : GPS receiver gives altitude output, but it has ~0.4 seconds of delay. In order of overcome this, pressure sensor was added.
- Low freq. : from GPS receiver
- High freq. : from pressure sensor

CF setup & flight data



$h_{\text{from Pressure Sensor}}$

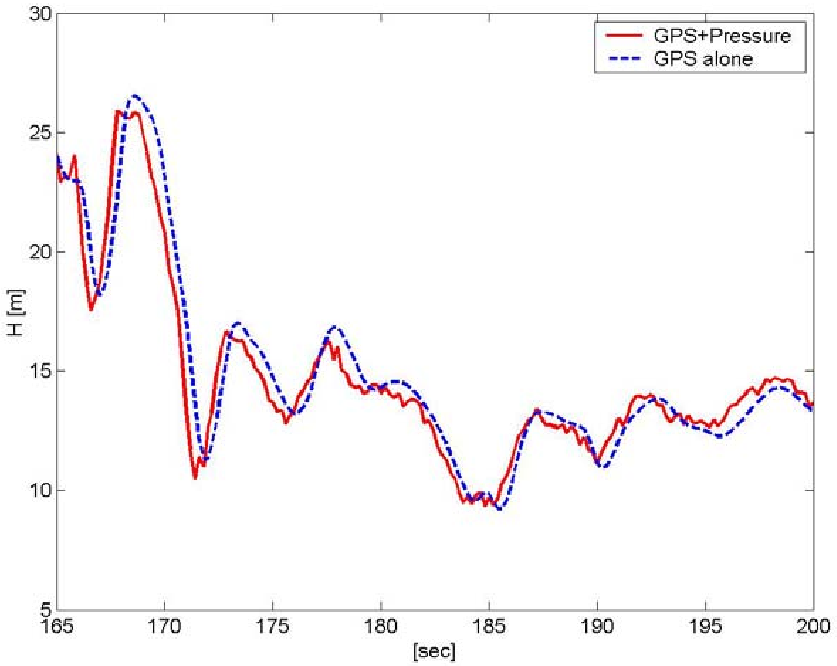
HPF

$h_{\text{from GPS KF}}$

LPF

+
+

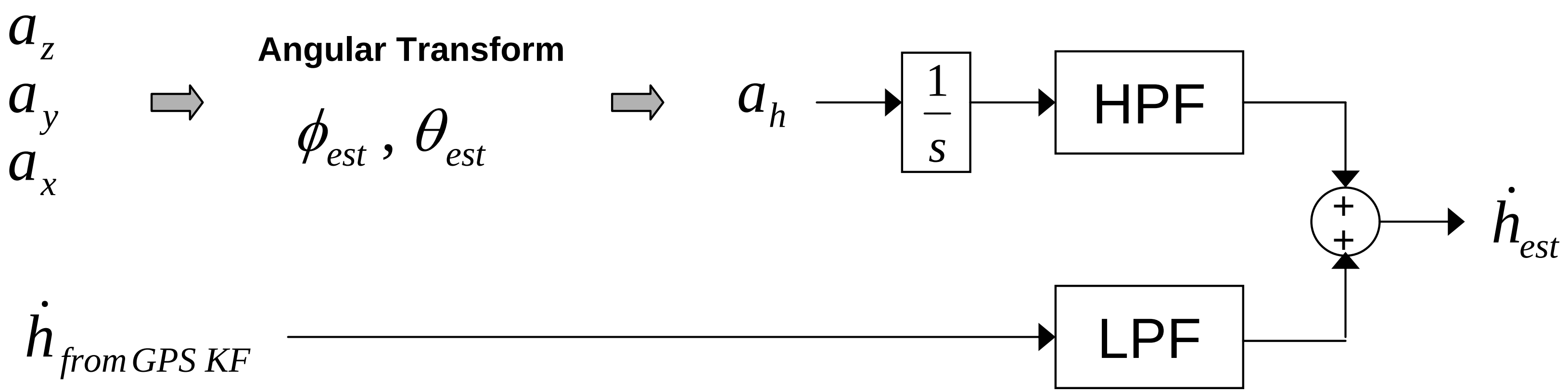
h_{est}



CF4. Altitude Rate Estimation

- Motivation : GPS receiver gives altitude rate, but it has ~0.4 seconds of delay.
  In order of overcome this, inertial sensor outputs were added.
- Low freq. : from GPS receiver
- High freq. : integrating acceleration estimate in altitude direction
  from inertial sensors

CF setup



$$\text{note}: \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix} - [\phi_{est}][\theta_{est}] \begin{Bmatrix} 0 \\ 0 \\ -g \end{Bmatrix}$$

$A_x\ , A_z$ – accelerometer outputs

$[\phi_{est}], [\theta_{est}]$: angular transformation matrices

Kalman Filter(KF) Examples

- KF1. Manipulation of GPS Outputs
- KF2. Removing Rate Gyro Bias Effect

KF 1. Manipulation of GPS Outputs

Background & Motivation

- Stand-alone GPS receiver gives position and velocity

- These are obtained by independent methods :
  - position ← pseudo-ranges
  - velocity ← Doppler effect

  and are certainly related $(\dot{x} = v)$

  → *Kalman filter can be used to combine them !*

- Motivation :

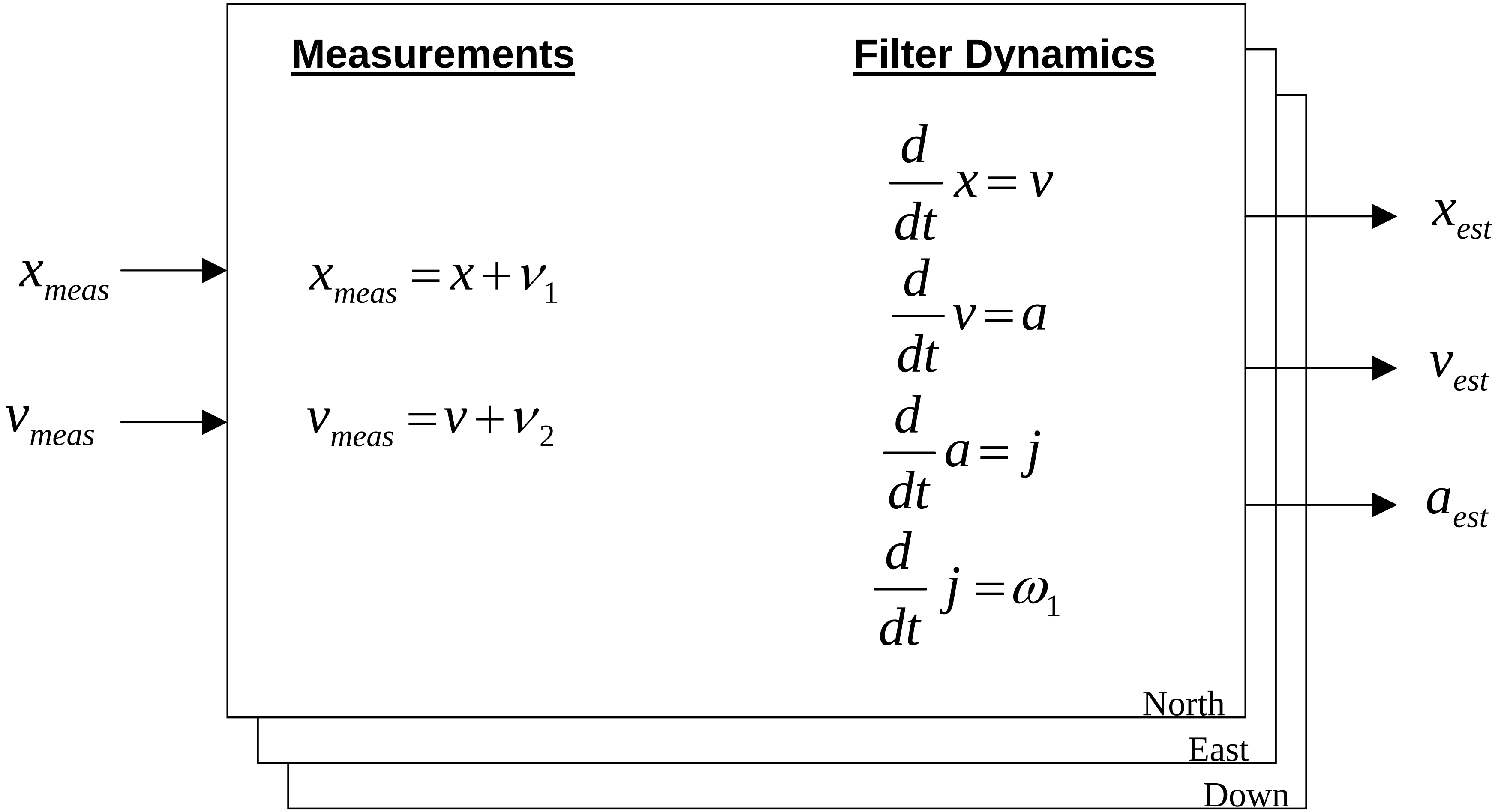
Typical Accuracies

Position	~ 30 m
Velocity	~ 0.15 m/s

Many GPS receivers provide high quality velocity information

→ *Use high quality velocity measurement to improve position estimate*

KF 1. Kalman Filter Setup



x_{meas}

v_{meas}

Measurements

$$x_{meas} = x + \nu_1$$

$$v_{meas} = v + \nu_2$$

Filter Dynamics

$$\frac{d}{dt} x = v$$

$$\frac{d}{dt} v = a$$

$$\frac{d}{dt} a = j$$

$$\frac{d}{dt} j = \omega_1$$

North

East

Down

x_{est}

v_{est}

a_{est}

x : position

v : velocity

a : acceleration

j : jerk

ν_i , ω_i : white noises

a_{est} :

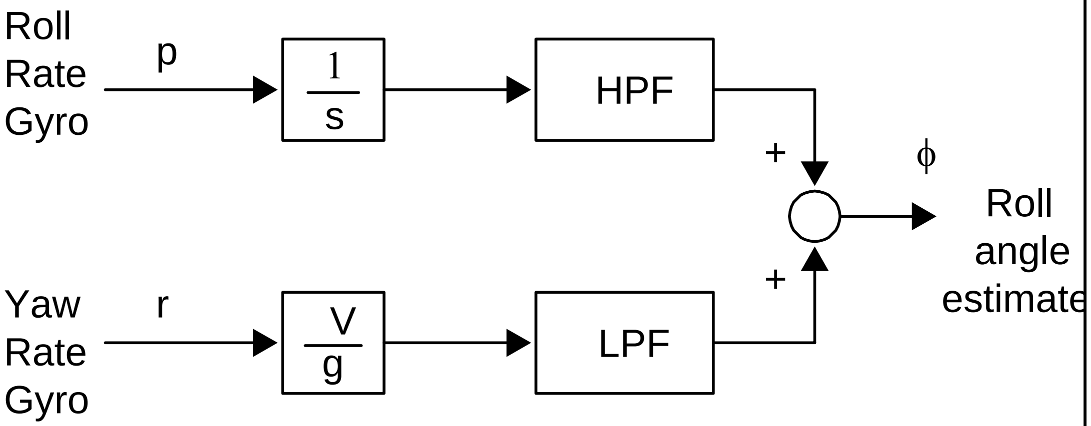
- noisy, but not biased
- combined with rate gyros in removing the gyro biases (KF2)

KF 2. Removing Rate Gyro Bias Effect

Background & Motivation

- In aircraft control, *roll angle* control is commonly used in inner-loop to create required *lateral acceleration* which is commanded from guidance outer-loop
- Biased roll angle estimate can cause steady-state error in cross-track

Complementary filter with roll & raw gyros (CF1)



Roll Rate Gyro

p

$\frac{1}{s}$

HPF

Yaw Rate Gyro

r

$\frac{V}{g}$

LPF

+

+

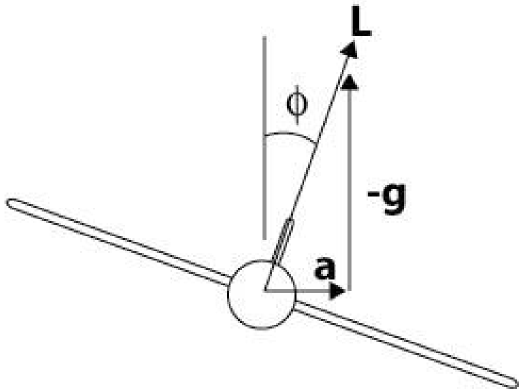
ϕ

Roll angle estimate

Drawback : biased estimate

Single-Antenna GPS Based Aircraft Attitude Determination

\- *Richard Kornfeld, Ph.D.* (1999)



Drawback : sampling rate limit (GPS), typical filter time constant ~ 0.5 sec.

$$a_s \approx g \cdot \phi \approx V \cdot r \qquad \dot{\phi} \approx p$$

KF 2. Kalman Filter Setup

from Rate Gyros

p_{meas} →

r_{meas} →

$(a_s)_{est}$ →

from GPS Kalman Filter

Measurement Equations

$$p_{meas} = p + bias_p + \nu_2$$

$$r_{meas} = \frac{g}{V}\phi + bias_r + \nu_3$$

$$a_s = g\phi + \nu_1$$

Filter Dynamics

$$\frac{d}{dt}\phi = p + \omega_1$$

$$\frac{d}{dt}p = \omega_2$$

$$\frac{d}{dt}bias_p = \omega_3$$

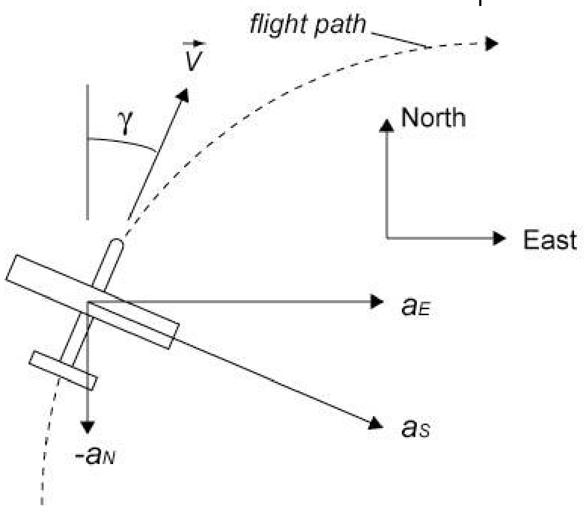
$$\frac{d}{dt}bias_r = \omega_4$$

→ ϕ_{est}

→ p_{est}

→ $(bias_p)_{est}$

→ $(bias_r)_{est}$



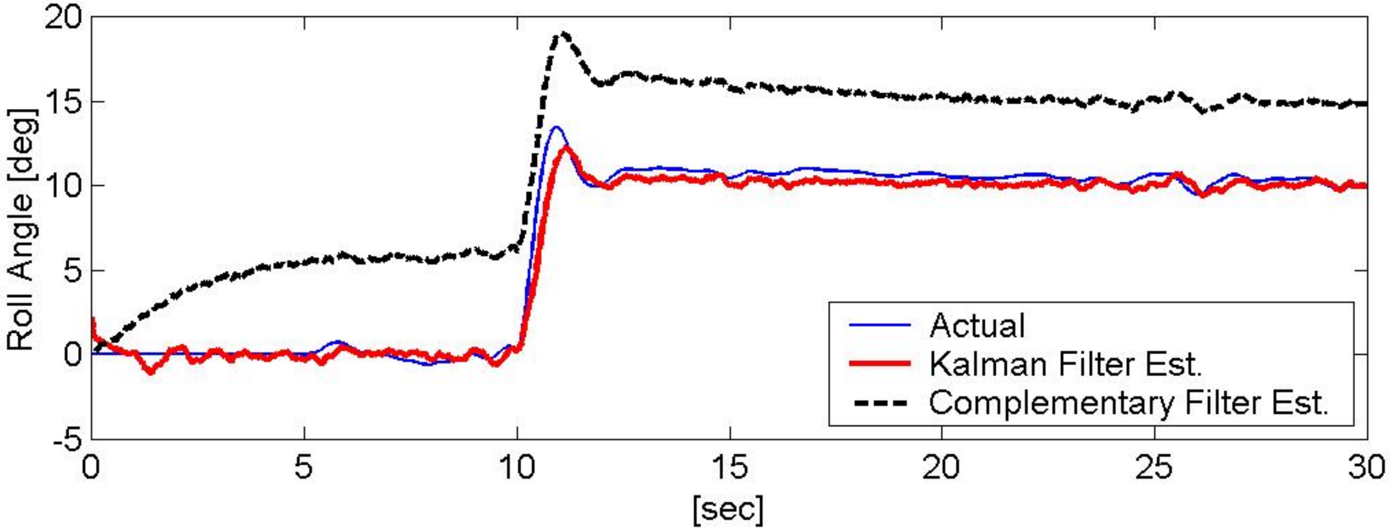
ϕ : bank angle V : velocity

a_s : acceleration in sideways direction

p : roll rate r : yaw rate

ν_i, ω_i : white noises

KF 2. Simulation Result



- Simulation for 10 degree bank angle hold
- Roll rate gyro bias=0.03 rad/s, yaw rate gyro bias = 0.02 rad/s were used in simulation

References

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