16.333: Lecture #1

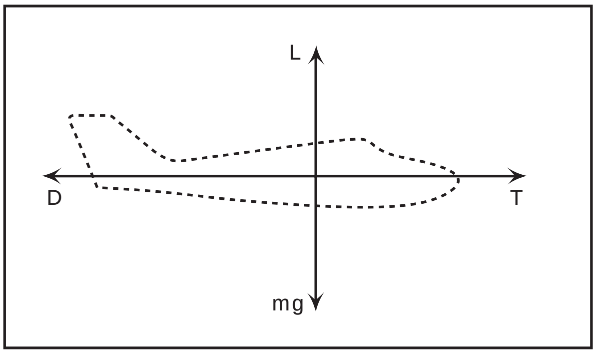
Equilibrium States

Aircraft performance

Introduction to basic terms

Aircraft Performance

- Accelerated horizontal flight - balance of forces
  - Engine thrust T
  - Lift L ($\perp$ to V)
  - Drag D ($\|$ to V)
  - Weight W

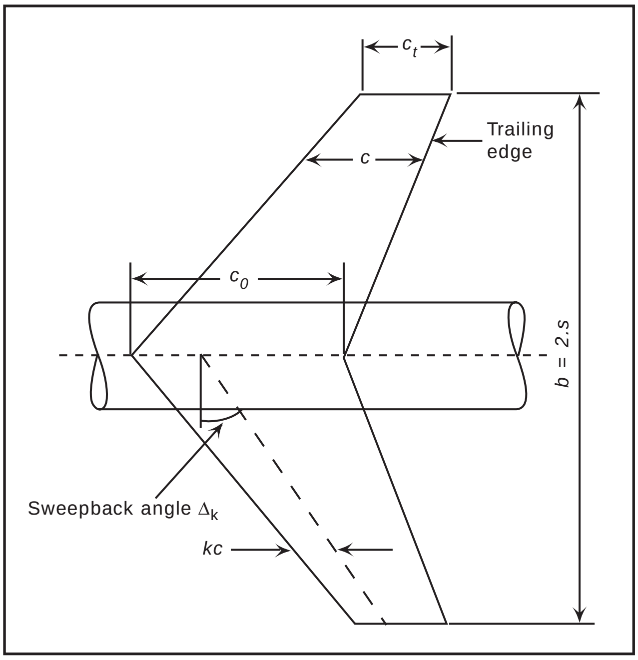


$$T - D = m\frac{dV}{dt} = 0 \text{ for steady flight}$$

and

$$L - W = 0$$

- Define $L = \frac{1}{2}\rho V^2 S C_L$ where
  - ρ – air density (standard tables)
  - S – gross wing area $= \bar{c} \times b$,
  - $\bar{c}$ = mean chord
  - b = wing span
  - $\mathcal{AR}$ – wing aspect ratio $= b/\bar{c}$



  - $Q = \frac{1}{2}\rho V^2$ dynamic pressure
  - V = speed relative to the air

- C_L lift coefficient – for low Mach number, $C_L = C_{L_\alpha}(\alpha - \alpha_0)$
  - ◇ α angle of incidence of wind to the wing
  - ◇ α_0 is the angle associated with zero lift

- Back to the performance:

$$L = \frac{1}{2}\rho V^2 S C_L \text{ and } L = mg$$

which implies that $V = \sqrt{\frac{2mg}{\rho S C_L}}$ so that

$$V \propto C_L^{-1/2}$$

and we can relate the effect of speed to wing lift

- A key number is stall speed, which is the lowest speed that an aircraft can fly steadily

$$V_s = \sqrt{\frac{2mg}{\rho S C_{L_{\max}}}}$$

where typically get $C_{L_{\max}}$ at $\alpha_{\max} = 10°$

Steady Gliding Flight

- Aircraft at a steady glide angle of γ

- Assume forces are in equilibrium

$$L - mg\cos\gamma = 0 \quad (1)$$
$$D + mg\sin\gamma = 0 \quad (2)$$

Gives that

$$\tan\gamma = \frac{D}{L} \equiv \frac{C_D}{C_L}$$

$\Rightarrow$ Minimum gliding angle obtained when C_D/C_L is a minimum

– High L/D gives a low gliding angle

- Note: typically

$$C_D = C_{D_{\min}} + \frac{C_L^2}{\pi A\!\!\!R e}$$

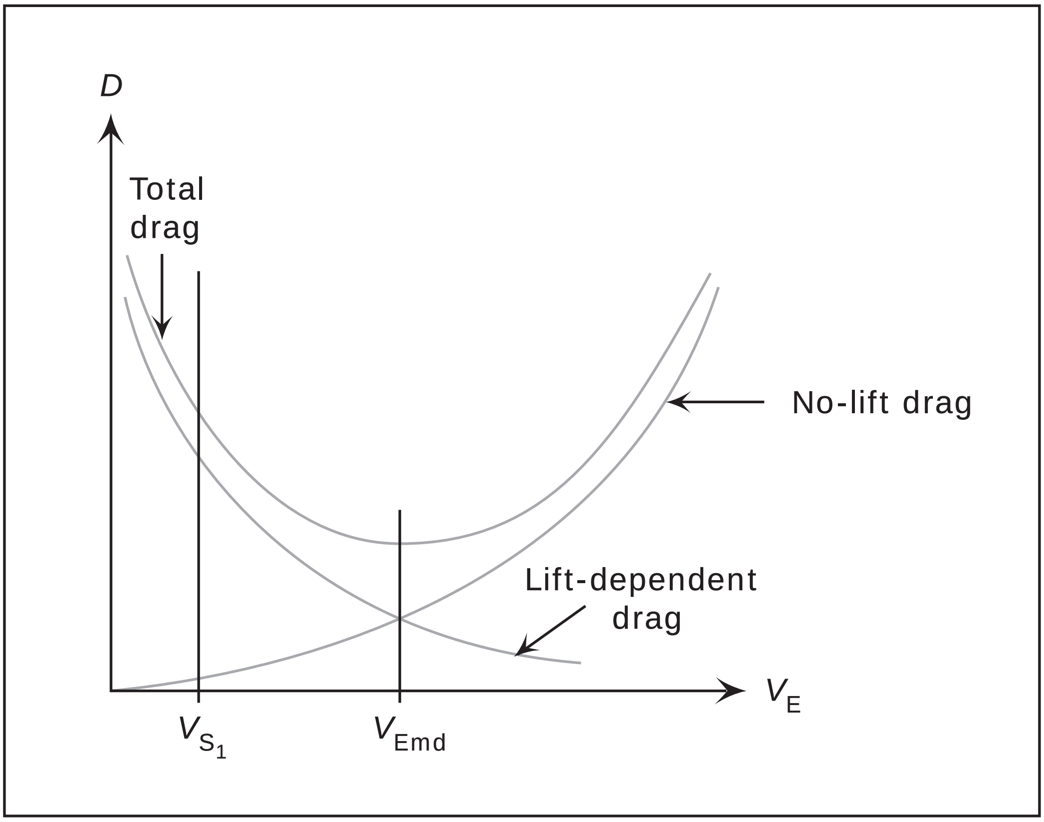
where

– $C_{D_{\min}}$ is the zero lift (friction/parasitic) drag
– C_L^2 gives the **lift induced drag**
– e is Oswald's **efficiency factor** $\approx 0.7 - 0.85$

- Total drag then given by

$$D = \frac{1}{2}\rho V^2 S C_D = \frac{1}{2}\rho V^2 S \left(C_{D_{\min}} + kC_L^2\right) \quad (3)$$

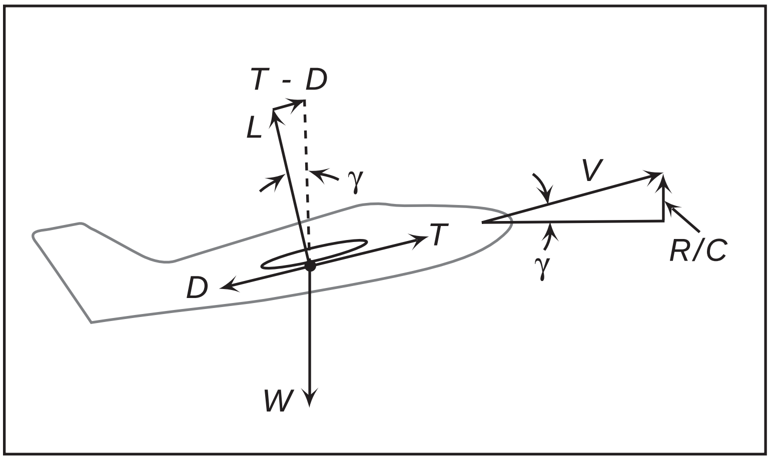
$$= \frac{1}{2}\rho V^2 S C_{D_{\min}} + k\frac{(mg)^2}{\frac{1}{2}\rho V^2 S} \quad (4)$$



- So that the speed for minimum drag is

$$V_{\min\ \text{drag}} = \sqrt{\frac{2mg}{\rho S}}\left(\frac{k}{C_{D_{\min}}}\right)^{1/4}$$

Steady Climb



- Equations:

$$T - D - W\sin\gamma = 0 \quad (5)$$

$$L - W\cos\gamma = 0 \quad (6)$$

$\Rrightarrow$ which gives

$$T - D - \frac{L}{\sin\gamma}\cos\gamma = 0$$

so that

$$\tan\gamma = \frac{T-D}{L}$$

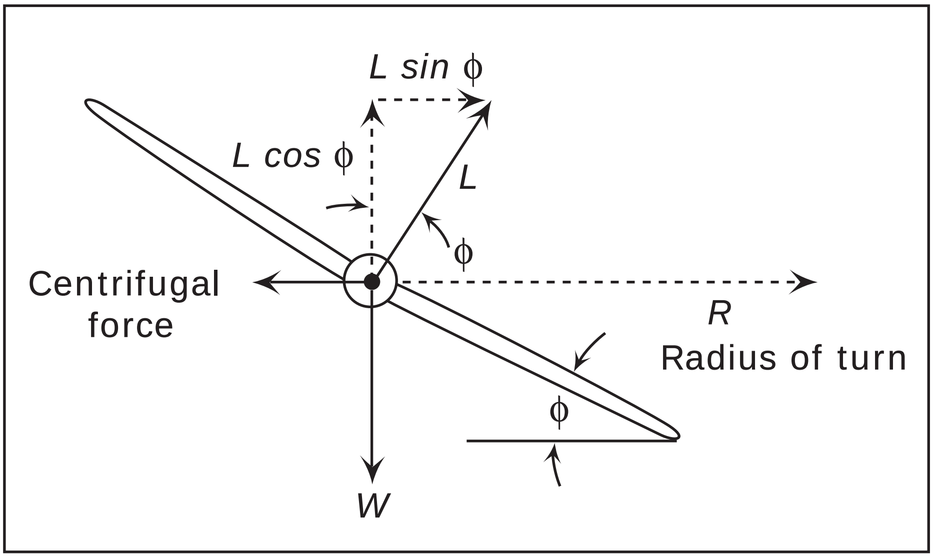
- Consistent with 1–3 if $T = 0$ since then γ as defined above is negative

- Note that for small γ, $\tan\gamma \approx \gamma \approx \sin\gamma$

$$R/C = V\sin\gamma \approx V\gamma \approx \frac{(T-D)V}{L}$$

so that the rate of climb is approximately equal to the excess power available (above that needed to maintain level flight)

Steady Turn



- Equations:

$$L \sin \phi = \text{centrifugal force} \tag{7}$$
$$= \frac{mV^2}{R} \tag{8}$$

$$L \cos \phi = W = mg \tag{9}$$

$$\Rightarrow \tan \phi = \frac{V^2}{Rg} \overset{V=R\omega}{=} \frac{V\omega}{g} \tag{10}$$

- Note: obtain $R_{\min}$ at $C_{L_{\max}}$

$$R_{\min}(\frac{1}{2}\rho V^2 S C_{L_{\max}}) \sin \phi = \frac{WV^2}{g}$$

$$\Rightarrow R_{\min} = \frac{W/S}{1/2 \rho g C_{L_{\max}} \sin \phi_{\max}}$$

where W/S is the wing loading and $\phi_{\max} < 30°$

- Define load factor $N = L/mg$. i.e. ratio of lift in turn to weight

$$N = \sec\phi = (1 + \tan^2\phi)^{1/2} \quad (11)$$

$$\tan\phi = \sqrt{N^2 - 1} \quad (12)$$

so that

$$R = \frac{V^2}{g\tan\phi} = \frac{V^2}{g\sqrt{N^2-1}}$$

- For a given load factor (wing strength)

$$R \propto V^2$$

- Compare straight level with turning flight
  - If same light coefficient

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{mg}{\frac{1}{2}\rho V^2 S} \equiv Nmg\frac{1}{2}\rho V_t^2 S$$

so that $V_t = \sqrt{N}V$ gives the speed increase (more lift)

- Note that C_L constant $\Rightarrow$ C_D constant $\Rightarrow$ $D \propto V^2 C_D$

$$\Rightarrow T_t \propto D_t \propto V_t^2 C_D \sim ND$$

so that must increase throttle or will descend in the turn