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(a) $\frac{F_y}{m} = 0 = \dot{V} + RU - PW$

$\frac{F_z}{m} = 0 = \dot{W} + PV - QU$

by assumption (c)

For small perturbation

$U = U_0 + u \qquad P = P_0 + p$

$V = v \qquad Q = q$

$W = w \qquad R = r$

$$\Rightarrow \begin{cases} \dot{v} + rU_0 - P_0 w = 0 & \cdots (1) \\ \dot{w} + P_0 v - U_0 q = 0 & \cdots (2) \end{cases}$$

Also, $M = I_{yy}\dot{Q} + PR(I_{xx} - I_{zz}) + (P^2 - R^2) I_{xz}$

$N = I_{zz}\dot{R} - I_{xz}\dot{P} + PQ(I_{yy} - I_{xx}) + QRI_{xz}$

From assumption

$I_{xz} \approx 0, \quad M = -K_w w, \quad N = K_v v$

$$\Rightarrow \begin{cases} I_{yy}\dot{q} + P_0 r(I_{xx} - I_{zz}) + K_w w = 0 & \cdots (3) \\ I_{zz}\dot{r} + P_0 q(I_{yy} - I_{xx}) - K_v v = 0 & \cdots (4) \end{cases}$$

From the abscence of rolling ($P_0 = 0$)

$(3) \rightarrow I_{yy}\dot{q} + K_w w = 0 \xrightarrow{\frac{d}{dt}} I_{yy}\ddot{q} + K_w \dot{w} = 0$

$(2) \rightarrow \dot{w} - U_0 q = 0$

$\Rightarrow I_{yy}\ddot{q} + K_w U_0 q = 0 \qquad \therefore \underline{\omega_y^2 = \frac{K_w U_0}{I_{yy}}}$