

16.333 Homework Assignment #1

1. Consider a glider flying in a vertical plane at an angle γ to the horizontal. As we discussed in class, for a glider $\gamma < 0$. In a steady glide, the force balance is given by:

$$D + W \sin \gamma = 0 \quad (1)$$

$$L - W \cos \gamma = 0 \quad (2)$$

The kinematic equations for the system give us that the distance covered along the ground satisfies $\dot{x} = V \cos \gamma$ and $\dot{h} = V \sin \gamma$.

For small γ we showed that the flight velocity that gives the flattest glide is given by

$$V_{fg} = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{K}{C_{D0}}}$$

where $C_D = C_{D0} + KC_L^2$.

- (a) Show that, for a given height, the distance covered with respect to the ground satisfies

$$\frac{dx}{dh} = \frac{1}{\gamma} = -E \Rightarrow R = E\Delta h$$

where $E = C_L/C_D$ and R is the range (assume a constant angle of attack so that E is constant during the glide). What speed does this suggest we should glide at to maximize the range?

- (b) Now consider the sink rate $\dot{h}_s = -\dot{h}$. Show that

$$\dot{h}_s \approx -V\gamma \approx \frac{DV}{W} = \sqrt{\frac{2W}{\rho S}} \frac{C_D}{C_L^{3/2}}$$

This suggests that the sink rate is a minimum when $\frac{C_D}{C_L^{3/2}}$ is a minimum. Show that this corresponds to a flight speed of

$$V_{ms} = \sqrt{\frac{2W}{\rho S}} \sqrt[4]{\frac{K}{3C_{D0}}} \approx 0.76V_{fg}$$

(Hint: find the C_L that minimizes $\frac{C_D}{C_L^{3/2}}$).

Discussion: The conclusion from this analysis is that to maximize range you fly at the speed that minimizes drag but slightly slower than that to minimize sink rate. Sailplane pilots tend to fly at V_{ms} when in “lift” mode, i.e. find an upward gust and then accelerate to V_{fg} when the updraft dies and they what to cover the most ground looking for another “lift”.

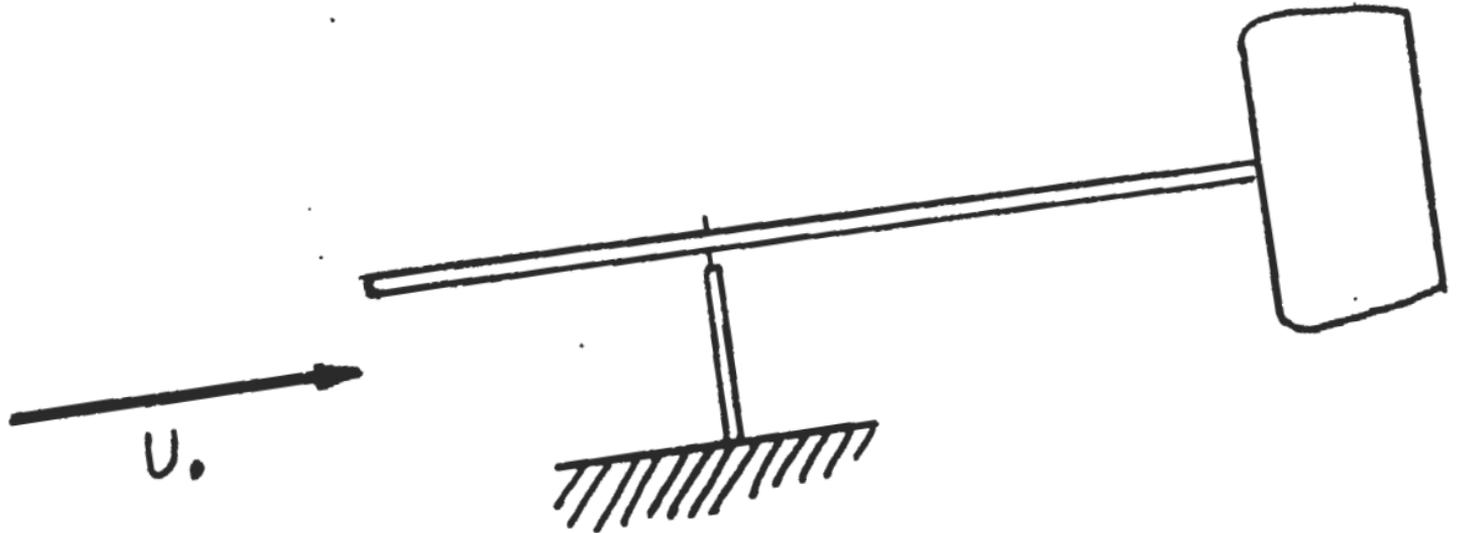
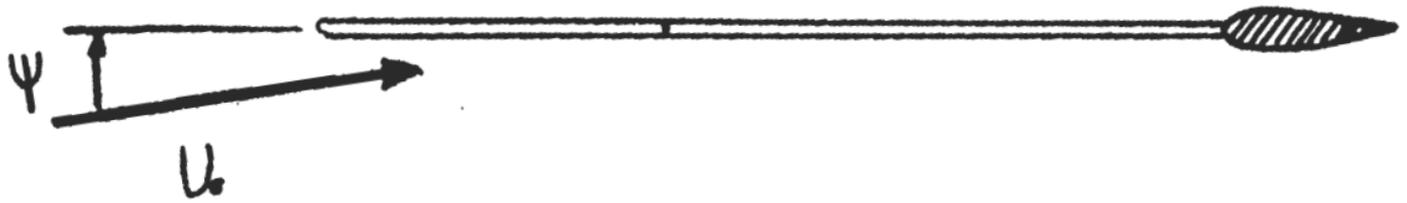
2. Consider the “weathervane” shown in the figure which allows us to explore some of the basics associated with aircraft motion. The aerodynamic surface (tail) acts like a wing, with a *lift* force proportional to the angle of attack α . The drag of this wing can safely be ignored.

See Next Page

Figure 1: Test Configuration for Problem #2.

- (a) When the arm is moving with rate $\dot{\Psi}$, find the effective angle of attack for the tail.
- (b) Assuming small angles, linearize and find the differential equation that governs the dynamics of this system. Assume that the damping is added at the support post by a torsional spring and is proportional to $\dot{\Psi}$. The damping is less than the *critical* level.
- (c) What are the roots of this system when: (i) the weathervane is as shown, and (ii) the vane is turned into the wind so that the tail is at the front?
- (d) Qualitatively discuss the response of the weathervane to a 20 degree change in the wind direction (U_0).
- (e) Develop a simulink simulation of the full nonlinear equations and compare the response of the nonlinear and linear systems.

Damping $\propto \dot{\psi}$



3. (Adapted from Etkin and Reid, 2.5) The following data apply to a 1/25th scale wind tunnel model of a transport airplane. The full-scale mass of the aircraft is 22,680kg. Assume that the aerodynamic data can be applied to the full-scale vehicle. For level unaccelerated flight at $V=239$ knots (123 m/s) of the full-scale aircraft, under the assumption that propulsion effects can be ignored,
- Find the limits on the tail angle i_t and CG position h imposed by the conditions C_{m_0} and $C_{m_\alpha} < 0$.
 - For trimmed flight with $\delta_e = 0$, plot i_t vs. h for the aircraft and indicate where this line meets the boundaries in part (a).

Geometric data

- wing area $S = 0.139\text{m}^2$ (so need to scale this are up by 25^2 – lengths scale by 25)
- wing mean aerodynamic chord $\bar{c} = 15.61\text{cm}$
- $l_t = 38.84\text{cm}$
- Tail area $S_t = 0.0342\text{m}^2$

Aerodynamic Data (does not need to scale)

- $C_{L_w} = 0.077/\text{deg}$ (convert to radians)
- $C_{L_t} = 0.064/\text{deg}$
- $\epsilon_0 = 0.72^\circ$
- $d\epsilon_0/d\alpha = 0.30$
- $C_{m_{acw}} = -0.018$
- $h_{\bar{n}} = 0.25$ (notes page 2–4)
- $\rho = 1.225\text{kg}/\text{m}^3$

4. Stability

- Draw a picture similar to those on 2–1 of the notes for a system that is statically stable, but not dynamically stable.
 - Consider a second order system such as a mass vibrating on a spring/damper.
 - The condition for static stability is that there be a restoring force when the system is displaced from equilibrium. What condition does that impose on the spring constant k ? What does that imply about the roots of the second order system?
 - What does the condition of dynamic stability imply about the damper constant c ? What does that condition imply about the roots of the system?
 - Do these conditions support the claim that static stability is a necessary but not sufficient condition for dynamic stability?
5. Download the Aerosim blockset discussed in the first class. Modify `navion_demo1` so that you can plot the altitude as a function of time.