16.30/31 December 2, 2010
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16.30/31 Homework Practice Problems #7

PRACTICE PROBLEMS ONLY - not to be submitted for credit.

Goals: Describing functions; Lyapunov stability analysis

1.

Problems 14.21 and 14.22 removed due to copyright restrictions.
Van de Vegte, John. *Feedback Control Systems*. 3rd ed.
Prentice Hall, 1993. ISBN: 9780130163790.

2.

3.

Problem 4.4 removed due to copyright restrictions.
Khalil, Hassan. *Nonlinear Systems*. 3rd ed. Prentice Hall, 2001. ISBN: 9780130673893.

4. Prove using the Lyapunov Theorem that the origin is a stable equilibrium for each of the following systems:

   (a) System 1:

$$\begin{aligned} \dot{x} &= -x^3 - y^2 \\ \dot{y} &= xy - y^3 \end{aligned}$$

   (b) System 2:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x^3 \end{aligned}$$

5. **(Challenge problem)** Consider the second-order nonlinear system

$$\begin{aligned} \dot{x_1} = -x_2 + \epsilon x_1 (x_1^2 + x_2^2) \sin(x_1^2 + x_2^2) \\ \dot{x_2} = x_1 + \epsilon x_2 (x_1^2 + x_2^2) \sin(x_1^2 + x_2^2). \end{aligned}$$

   Study the stability of the equilibrium at the origin, for $\epsilon \in [-1, 1]$. Is linearization sufficient? Find a Lyapunov function V that proves/disproves stability.

   Hint: In order to find V, it may be helpful to draw the trajectories of the system in the phase plane (x_1, x_2).

[1] H. K. Khalil. *Nonlinear Systems*. 3rd ed, Prentice Hall, 2002.