

16.30/31 Homework Assignment #5

Goals: Controllability and observability (part 2), full-state feedback, LQR, system zeros

1. For each state-space model, identify whether the system is *observable*, *controllable*, *detectable*, and/or *stabilizable*. Conclude whether each model is a minimal realization.

(a) $A = \begin{bmatrix} -1 & 3 \\ 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $C = [2 \quad -1]$, $D = [1 \quad 0]$

(b) $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -3 & 0 \\ -1 & 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $C = [1 \quad 0 \quad 0]$, $D = -1$

2. Consider the state-space model

$$\dot{\mathbf{x}} = \begin{bmatrix} -5 & 1 \\ 14 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$
$$y = [1 \quad 0] \mathbf{x}.$$

Suppose we want to apply full-state feedback to the system, of the form $u = -K\mathbf{x}$.

- (a) Without computing the transfer function, identify the open-loop system poles and zeros.
 - (b) Select K such that the closed-loop system poles are placed at the roots of
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0.$$
 - (c) Use your analytic expression for K to support the following claim: “To move the poles a long way takes large gains.”
 - (d) Select K such that the closed-loop system poles are placed at $s = -9$ and $s = -4$.
3. Consider a satellite whose thrusters are aligned with the coordinate axes; we can then decouple the satellite’s dynamics along each coordinate direction. The simplest model of the dynamics in each direction is

$$F = M\ddot{x},$$

where F is the net thruster force applied (in N), $M \geq 0$ is the satellite mass (in kg), and x is the displacement in that direction (in m). If only the satellite displacement x can be measured and not the satellite velocity v , then

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix} F$$
$$y = [1 \quad 0] \mathbf{x}$$

is a valid state-space model for this system, where $\mathbf{x} = [x \quad v]^T$.

- (a) Suppose we want to apply full-state feedback to the system, of the form $F = -K\mathbf{x}$. Find K such that the cost function

$$J = \int_0^{\infty} (x^2 + \rho^2 F^2) dt$$

is minimized, where $\rho > 0$ is some parameter. Your answer should be in terms of ρ and M .

- (b) What happens to your feedback gains as ρ is varied? Why does this make sense?
- In parts (c) through (e), assume that $\rho = 1$ and $M = 8$ kg.

(c) Confirm that the optimal feedback strategy from part (a) is $F = -x - 4v$.

(d) Suppose that at time $t = 0$ s, the satellite displacement is $x(0) = 0$ m and the satellite velocity is $v(0) = 10$ m/s. What will be the displacement and velocity at time $t = 5$ s if:

- No feedback is applied? (open-loop)
- The optimal feedback from part (c) is applied? (closed-loop)

You may use `expm.m` in Matlab, but all other work must be done by hand.

(e) Using Matlab, plot the closed-loop state response to the initial condition $\mathbf{x}(0) = [0 \ 10]^T$. (This should match your answers in part (d) at time $t = 5$ s.)

4. (16.31 required/16.30 extra credit) We have talked at length about selecting the full-state feedback gains to change the pole locations of the system, but we have not discussed how to vary the locations of the system zeros. Indeed, as mentioned in lecture, it can be shown that:

When full state feedback is used ($u = \bar{N}r - Kx$) to control a system, the zeros remain unchanged by the feedback.

Confirm that this statement is true by analyzing the zero locations for the closed-loop system, which are given by the roots of the polynomial:

$$\det \begin{bmatrix} sI - (A - BK) & -\bar{N}B \\ C & 0 \end{bmatrix} = 0$$

The best way to proceed is to show that, through a series of transformations that preserve whether or not the determinant is zero, you can get the following reduction:

$$\det \begin{bmatrix} sI - (A - BK) & -\bar{N}B \\ C & 0 \end{bmatrix} = 0 \Rightarrow \det \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix} = 0$$

which, of course, is the same polynomial used to find the open-loop zeros of the system.

You only need to prove this for the case where $\begin{bmatrix} sI - (A - BK) & -\bar{N}B \\ C & 0 \end{bmatrix}$ is square;

I may give you a few extra credit points if you prove it in general.

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