16.30/31 October 15, 2010
Prof. J. P. How and Prof. E. Frazzoli **Due:** October 22, 2010
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16.30/31 Homework Assignment #4

Goals: Modal analysis, transfer matrices, controllability and observability (part 1), linear system theory

1. Consider the system with two states, and the state-space model matrices given by:

$$A = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ K \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where $K \in \mathbb{R}$ is a parameter to be specified.

(a) Find the transfer function $G(s)$ for the system. Discuss the structure of $G(s)$ for various values of K.

(b) Form the observability matrix for the system. Is the system observable for all values of K?

(c) Form the controllability matrix for the system. Is the system controllable for all values of K?

(d) Compare your observations in parts (b) and (c) with those in part (a).

2. Given the transfer function from input $u(t)$ to output $y(t)$,

$$\frac{Y(s)}{U(s)} = \frac{s^2 - 4s + 3}{(s^2 + 6s + 8)(s^2 + 25)}$$

(a) Develop a state space model for this transfer function, in the standard form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du. \end{aligned}$$

(b) Suppose that zero input is applied, such that $u = 0$. Perform a modal analysis of the state response for this open-loop system. Your analysis should include the nature of the time response for each mode, as well as how each element of the state vector $x = \begin{bmatrix} x_1 \cdots x_n \end{bmatrix}^T$ contributes to that mode. Which mode dominates the time response? You may use Matlab to assist in your analysis.

(c) Now suppose that input of the form $u = Ky$ is applied, where $K = -15$. Repeat the modal analysis of part (b) for this closed-loop system. (We will talk much more about this type of feedback later in the course.)

3. Given the MIMO system,

$$G(s) = \begin{bmatrix} \dfrac{7}{s+2} & \dfrac{2s+8}{s^2+5s+6} \\ \dfrac{3s+15}{s^2+7s+10} & \dfrac{5}{s+3} \end{bmatrix}$$

(a) Develop a state space model using the technique described at the bottom of page 8–5. Using Matlab, verify that this is not a minimal realization.

(b) Develop a state space model using Gilbert's realization method on page 8–8. Using Matlab, verify that this is a minimal realization.

Hint: It is easy to confirm that each state space model will give the same transfer function matrix.

4. (16.31 required/16.30 extra credit) Consider the homogeneous system

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t)$$

with initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$. The general solution to this differential equation is given by

$$\mathbf{x}(t) = \Phi(t, t_0)\mathbf{x}(t_0)$$

where $\Phi(t_1, t_1) = I$. Prove that the following properties of the state transition matrix are true:

(a) $\Phi(t_2, t_0) = \Phi(t_2, t_1)\Phi(t_1, t_0)$, regardless of the order of the t_i

(b) $\Phi(t, \tau) = \Phi(\tau, t)^{-1}$

(c) $\frac{\partial}{\partial t}\Phi(t, t_0) = A(t)\Phi(t, t_0)$

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