Final examination
16.30 Control Systems S.H.O.
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100 points total

1.[8pt] Increased track densities for computer disk drives require careful design of the head positioning control. The transfer function of the system is

$$G(s) = \frac{K}{(s+1)^2}$$

Plot the Nyquist plot for this system when K=4. Compute the phase and magnitude at 0.5, 1, 2, 4 radians/sec.

2. [10pt] A feedback control system has a loop transfer function

$$G(s)H(s) = \frac{50}{s^2 + 11s + 10}.$$

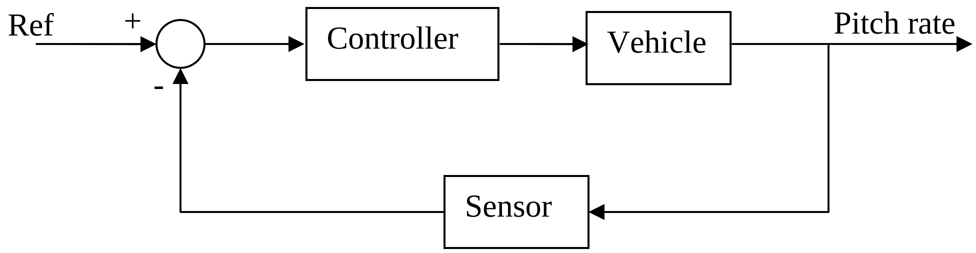
a/ Determine the corner frequencies (beak points) for the Bode Plot.
b/ What is the slope of the Bode plot at very low and very high frequencies?
c/ Sketch the Bode plot of the system (magnitude and phase).

3. [10pt] The experimental Oblique Wing Aircraft has a wing that pivots about an axis parallel to the yaw axis (horizontal pivoting). The wing is in normal, unskewed position for low speeds and can move to a skewed position for improved supersonic flight. The loop gain for the longitudinal control system is

$$G(s) = \frac{4(0.5s+1)}{s(2s+1)\left[\left(\frac{s}{8}\right)^2 + \left(\frac{s}{20}\right) + 1\right]}$$

a/ Determine the Bode diagram.
b/ Find the frequency when the magnitude is 0dB, and the frequency when the phase is -180deg. What are the gain and phase margins?

4. [10pt] The Space Shuttle uses elevons at the trailing edge of the wing and a brake on the tail to control its flight. The block diagram of a pitch rate control system for the shuttle is given below



The sensor is represented by a gain $H(s)=0.5$, and the vehicle is represented by the transfer function

$$G(s) = \frac{0.30(s+0.05)(s^2+1600)}{(s^2+0.05s+16)(s+70)}.$$

The controller can be a gain or any suitable transfer function.
a/ Draw the Bode diagram of the system when the controller is $K(s)=2$ and determine the stability margin.
b/ Draw the Bode diagram of the system when
$K(s)=K_1+K_2/s$ and $K_2/K_1=0.5$.

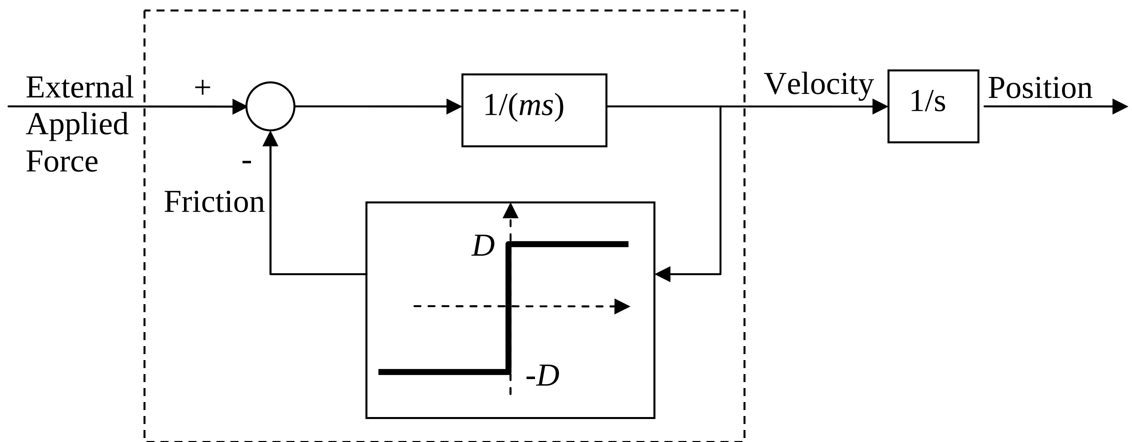
The gain K_1 should be selected so that the gain margin is 10 dB.

5/ [14pt] The roll axis of a high performance jet airplane is given by the transfer function

$$G(s) = \frac{2}{(s+10)(s^2+2s+2)}.$$

This is the transfer function from aileron to roll angle. Design a closed-loop controller such that the step response is well-behaved and the steady-state error to that step response is zero.

6/ [24pt] Dry (or Coulomb) friction applied to a mass m sliding on a surface may be modeled as a nonlinear element with a feedback loop as given below.



In this diagram, the dry friction maximum amplitude is *D*. For all numerical calculations, assume *D*=1 and *m*=1.

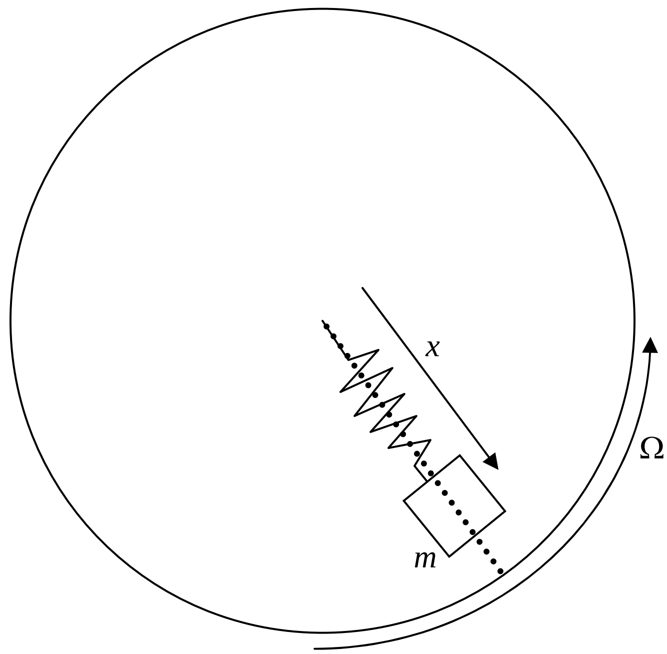
a/ Check and show that the above diagram indeed matches the intuitive notion of dry friction:

- When the mass is at rest (velocity zero), the friction force counteracts any force of amplitude less than *D* and keeps the mass still.
- When the mass is moving, the friction is of amplitude *D* and opposite to the motion of the system.
- Can the system inside the dashed box undergo uncontrolled oscillations?

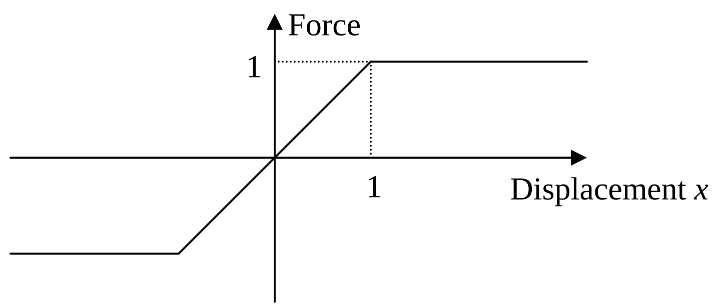
b/ Give a qualitative description of the steady-state velocity response of the system to a sinusoidal input force of amplitude 2*D*.

c/ Compute the describing function of the dynamic nonlinear element contained in the dashed box. Is it dependent on amplitude? Is it dependent on frequency?

7.[24pt]



A mass *m* is mounted on a disk in such a way that it can slide (with no friction) only along a groove (dashed line) rigidly attached to the disk. This mass is connected to the center of the disc by a softening spring. The characteristic of the spring is given in the figure below; it is symmetric with respect to the origin:



The disc can be rotated at any fixed, desired angular rate Ω. We assume the mass *m* is one.

a/ Assume Ω is zero. Plot the phase-plane characteristic of the system (velocity d*x*/d*t* vs. position *x*).

b/ Assume now Ω=0.1 rad/sec. In addition to the restoring force of the spring, the mass is now also subject to a centrifugal force of amplitude $mx\Omega^2$. Plot the phase-plane for the same system.

c/ Give a qualitative description of the evolution of the phase-plane for this system as the parameter Ω increases. Take a good look at the different equilibrium points. How do they move around?