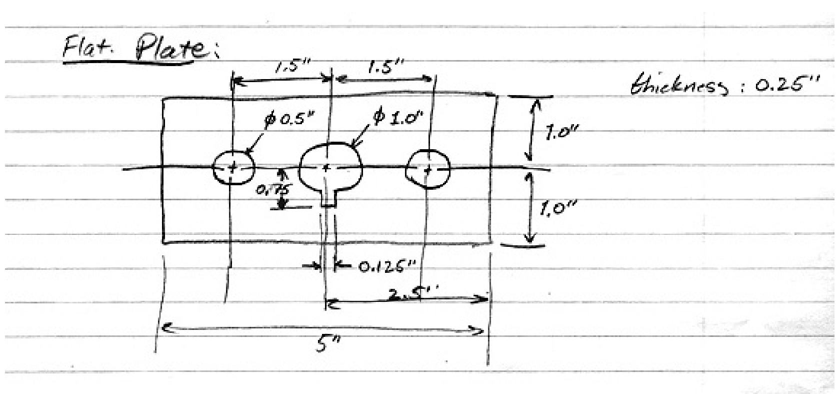
16.21 - Techniques of structural analysis and design
Homework assignment # 3
Handed out: 2/25/05
Due: 3/4/05

February 24, 2005

Warm-up exercises (not for grade)

- Problem 3.23 from textbook
- Problem 3.24 from textbook
- Problem 3.25 from textbook
- (Compliments of C. Graff.) Create a solid model of the flat plate in the figure using Solidworks (you may turn in your file electronically for feedback purposes).



Problems for grade

1. Problem 3.26 from textbook
2. Problem 3.27 from textbook:
   (a) Find the linear strains corresponding to the following displacement field:

$$\begin{aligned} u_1 &= u_1^0(x_1, x_2) + x_3\phi_1(x_1, x_2) \\ u_2 &= u_2^0(x_1, x_2) + x_3\phi_2(x_1, x_2) \\ u_3 &= u_3^0(x_1, x_2) \end{aligned}$$

   (b) Verify that the resulting strain field is compatible for any choice of functions u_i^0, ϕ_i.
3. Problem 3.30 from textbook
4. Justify our step in the derivation of the local form of the first law of thermodynamics for deforming bodies where we assumed:

$$\sigma_{ij}\frac{\partial u_i}{\partial u_j} = \sigma_{ij}\epsilon_{ij}$$

   i.e., demonstrate that the double scalar product (full contraction) of a symmetric tensor $\mathbf{A} = \mathbf{A}^T$, with an arbitrary tensor $\mathbf{B}$ amounts to contracting $\mathbf{A}$ with the symmetric part of $\mathbf{B}$:

$$\mathbf{B}^{sym} = \frac{1}{2}\big(\mathbf{B} + \mathbf{B}^T\big)$$

   (Hint: Decompose $\mathbf{B}$ into its symmetric and antisymmetric parts and show that the contraction of a symmetric tensor $\mathbf{A}$ with the antisymmetric part of $\mathbf{B}$:

$$\mathbf{B}^{antisym} = \frac{1}{2}\big(\mathbf{B} - \mathbf{B}^T\big)$$

   is zero.
5. Obtain the relationships between the engineering elastic constants (E, ν) and the Lamé constants (λ_1, λ_2).