

# 16.21 - Techniques of structural analysis and design

## Homework assignment # 1

Handed out: 2/09/05

Due: 2/16/05

February 10, 2005

Only problems 2, 3 and 4 are part of the assessment (contribute to the grade). It is recommended that the assignment be done in Mathematica in its entirety and turned in electronically on the MIT Server.

1. **Mathematica practice problems.** Write a Mathematica notebook containing the following items:

- (a) A header in a text cell containing the Homework assignment number and your name. If possible try to use a Style Sheet of your choice in your Mathematica notebooks from now on.
- (b) (Not for grade) Multiplication of a  $3 \times 3$  matrix by a  $3 \times 1$  vector. Choose arbitrary symbols (not numerical values) as matrix and vector coefficients. Repeat for coefficients with arbitrary numeric values. Repeat one more time for the case of a  $5 \times 5$  matrix  $A$  and a  $5 \times 1$  vector  $v$  whose coefficients satisfy the following functional dependence on the row ( $i$ ) and column ( $j$ ) number:

$$A_{ij} = i + j; v_i = i(i + 1)$$

- (c) Verification of candidate solutions of a differential equation by direct differentiation. Given the differential equation governing

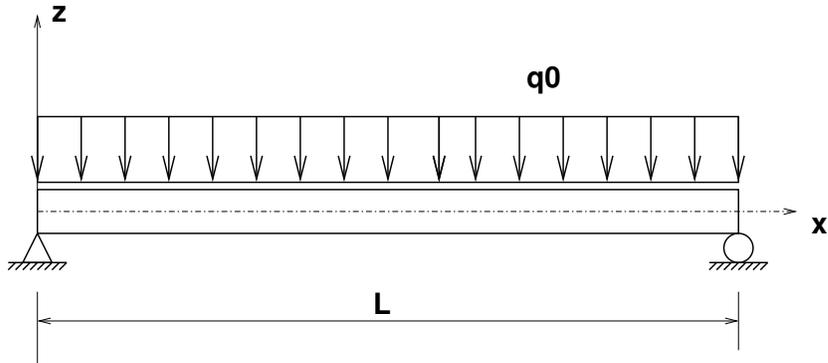
the elastic deflections of a beam due to a sinusoidal load:

$$EI \frac{d^4 w}{dx^4} + q_0 \sin \frac{\pi x}{L} = 0, \quad 0 < x < L$$

Evaluate if any of the following candidate functional forms provides a solution to this problem and, in that case, find the value of the constant(s). What boundary conditions are implied in the proposed solution?

- i.  $w(x) = A + Bx + Cx^2 + Dx^3$
- ii.  $w(x) = A \sin \frac{\pi x}{L}$
- iii.  $w(x) = A \cos \frac{\pi x}{L} + B \sin \frac{\pi x}{L}$

2. **Verification of that at equilibrium potential energy is a minimum in beam problem :**



Use mathematica to **evaluate** the potential energy of the beam in the figure for the following two functions in terms of their respective parameters ( $a$  and  $(a, b, c)$ , respectively). (Use the expression of the potential energy given in class):

1)

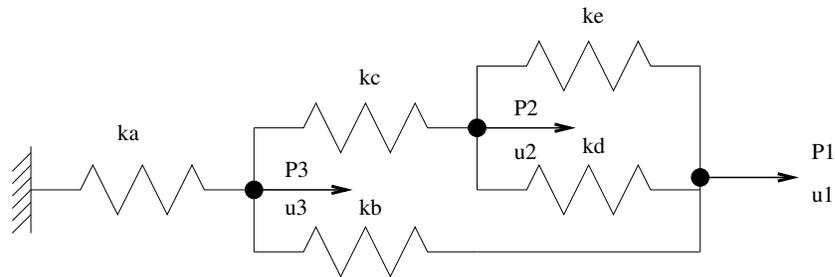
$$w_1(x) = a \frac{q_0 L^4}{EI} \frac{x}{L} \left(1 - \frac{x}{L}\right)$$

2)

$$w_3(x) = \frac{q_0 L^4}{EI} \left[ a \frac{x}{L} \left(1 - \frac{x}{L}\right) + b \frac{x}{L} \left(1 - \frac{x}{L}\right)^2 + c \frac{x}{L} \left(1 - \frac{x}{L}\right)^3 \right]$$

For each function, **plot** the potential energy vs. the value of  $a$  for a range  $a \in [-a_{min}, 3a_{min}]$ , where  $a_{min}$  is the value of  $a$  corresponding to the minimum potential energy given in class. Note: for the second function make  $b = b_{min}$  and  $c = c_{min}$ . **Draw your conclusions** about the behavior of the potential energy as a function of  $a$  for each case. (Extra: **Superimpose** the plots and draw your own conclusions by comparing them).

### 3. Energy approach applied to a spring system problem .



Find the displacements  $u_1, u_2, u_3$  of the nodes of the spring system in the figure in terms of the given applied nodal loads  $P_1, P_2, P_3$ . Use two different methods:

- Vector approach as learned in Unified Engineering (Hint: Apply “three great principles and label the internal forces in the springs as  $F_a, F_b, F_c, F_d, F_e$ ”). If you find this part too hard to be done in Mathematica, you can turn it in on paper.
- Energy method: Compute the elastic energy of each spring in terms of the displacements  $u_1, u_2, u_3$ , add up all the contributions to form the total “strain energy” of the system  $U$ . Compute the work done by each external force  $P_1, P_2, P_3$  on these displacements, add up all the contributions to form the total external work  $W$ . Form the total potential energy of the system as the difference between the “strain energy” and the work of the external forces:  $\Pi = U - W$ . Determine the equations of equilibrium and the solution by finding the stationary points of the total potential energy with respect to the displacements  $u_1, u_2, u_3$ . Comment on the advantages and disadvantages of this approach.

(c) Specialize the solution to the following two cases and provide your own interpretation of the results in both cases:

i.  $k_a = k_b = k_c = k_d = k_e = K$ ,  $P_1 = P_2 = 0$ ,  $P_3 = P$

ii.  $k_a = k_b = k_c = k_d = k_e = K$ ,  $P_2 = P_3 = 0$ ,  $P_1 = P$

4. Problem 3.15 from textbook. This problem can also be done on paper if you are not comfortable using Mathematica yet.