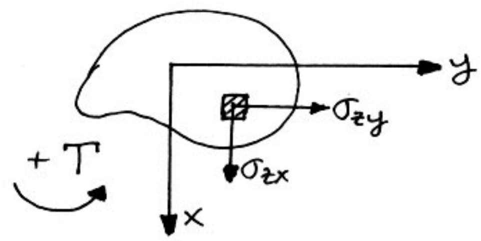
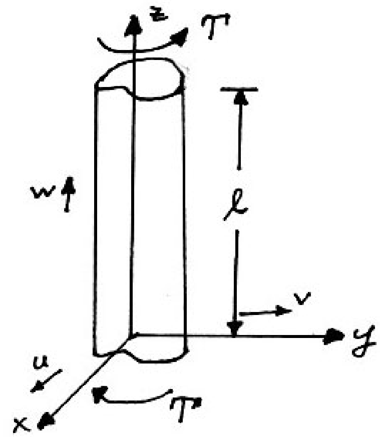
16.20 HANDOUT #3
Fall, 2002
Review of "Basic" Torsion Theory

SOLID CROSS-SECTIONS (*St. Venant Theory*)





$\phi = 0$ on contour (free boundary)

$$T = -2\iint \phi \, dxdy$$

$$\nabla^2\phi = 2Gk \qquad \frac{\partial\phi}{\partial x} = \sigma_{yz} \qquad \frac{\partial\phi}{\partial y} = -\sigma_{xz}$$

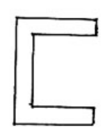
- Rigid rotation of cross-section
- Free to warp
- σ_{xz}, σ_{yz} only nonzero stresses

$$\frac{d\alpha}{dz} = \frac{T}{GJ}$$

GJ = torsional rigidity
J = torsion constant

Stress resultant = $\tau = \sqrt{\sigma_{zx}^2 + \sigma_{zy}^2}$

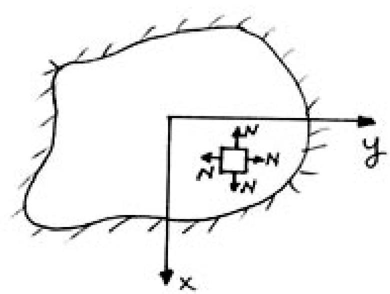
OPEN, THIN-WALLED SECTIONS (*Membrane Analogy*)



Same governing equation and B.C. for torsion and pressurized membrane

$$\nabla^2 w = -\frac{p_i}{N}$$

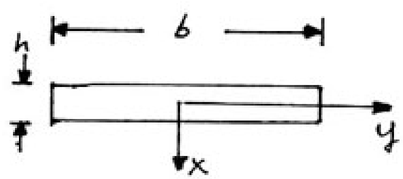
w = 0 on contour



Analogy:

Membrane		Torsion
w	$\rightarrow$	ϕ
p_i	$\rightarrow$	$-k$
N	$\rightarrow$	$\frac{1}{2G}$
$\frac{\partial w}{\partial x}$	$\rightarrow$	$\frac{\partial \phi}{\partial x} = \sigma_{zy}$
$\frac{\partial w}{\partial y}$	$\rightarrow$	$\frac{\partial \phi}{\partial y} = -\sigma_{zx}$
Volume $= \iint w\,dx\,dy$	$\rightarrow$	$-\frac{T}{2}$

Apply to a narrow rectangular cross-section



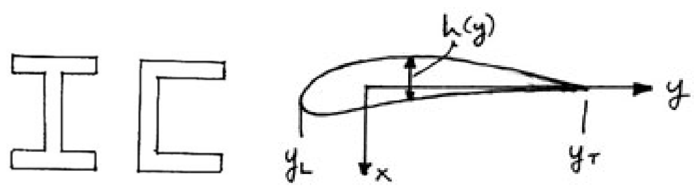
(local axes)

$$J = \frac{bh^3}{3}$$

$$\tau_{res} \approx \frac{2T}{J} x = \sigma_{yz}$$

$$(\sigma_{xz} = 0)$$

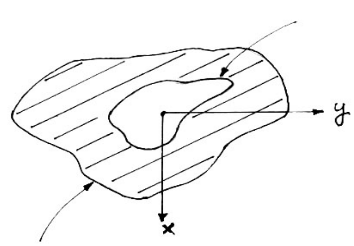
apply to:



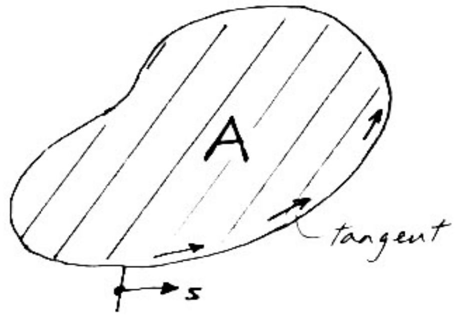
CLOSED, THICK-WALLED SECTIONS

$\phi = C_1$ on one boundary

$\phi = C_2$ on one boundary



$$\oint \tau ds = 2AGk \quad \text{on any closed boundary}$$

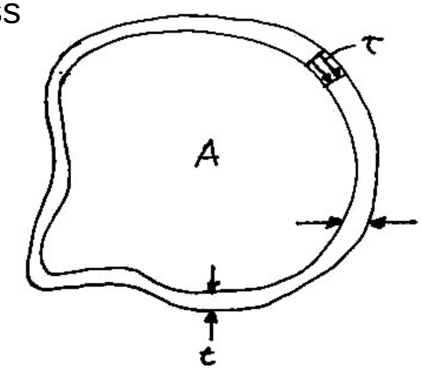


THIN-WALLED CLOSED SECTIONS

τ_{res} constant through thickness

$$\oint \tau ds = 2GKA$$

"shear flow": $q = \tau t$



A = enclosed area

Bredt's formula
$$\boxed{\tau_{\text{resultant}} = \frac{T}{2At}}$$

$$J = \frac{4A^2}{\oint \frac{ds}{t}}$$

Note: Free-to-warp assumptions violated near end constraints for all torsion problems.