Unit 19
General Dynamic Considerations

Reference: Elements of Vibration Analysis, Meirovitch, McGraw-Hill, 1975.

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VI. (Introduction to) Structural Dynamics

Thus far have considered only static response. However, things also move, this includes structures.

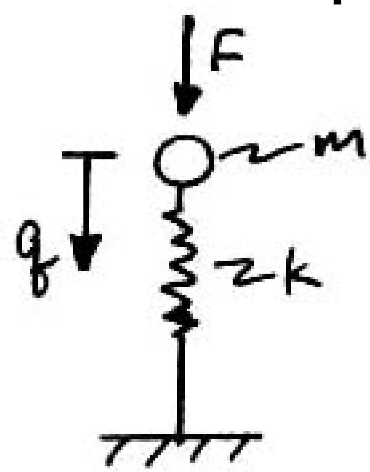
Can actually identify three "categories" of response:

A. (Quasi) - Static ["quasi" because the load must first be applied]
B. Dynamic
C. Wave Propagation

What is the key consideration in determining which regime one is in?
--> the frequency of the forcing function

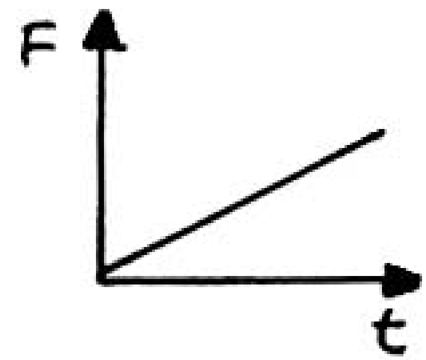
Example: Mass on a Spring

Figure 19.1 **Representation of mass on a spring**



A) Push <u>very</u> slowly

Figure 19.2 **Representation of force increasing slowly with time**

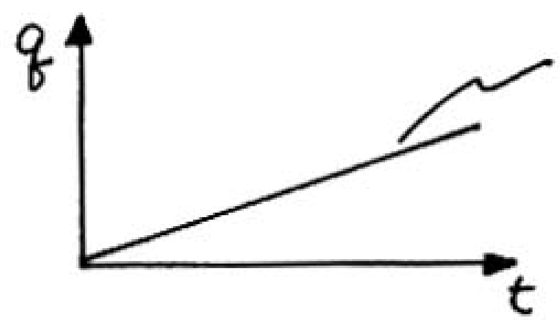


The response is basically determined by:

$$F = kq$$

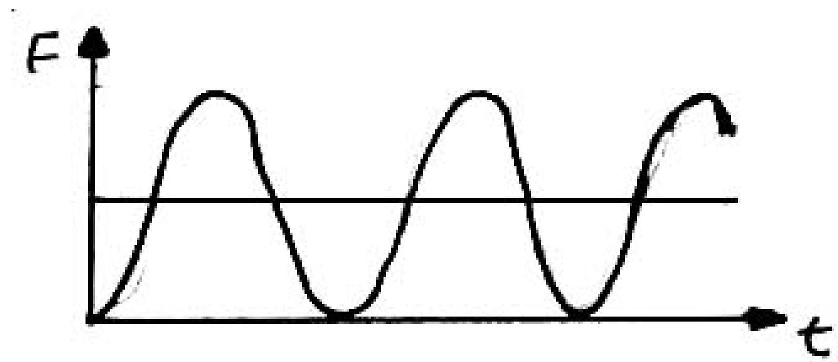
$$\Rightarrow q(t) = \frac{F(t)}{k} \approx \frac{F}{k}$$

Figure 19.3 **Deflection response versus time for mass in spring with loads slowly increasing with time**



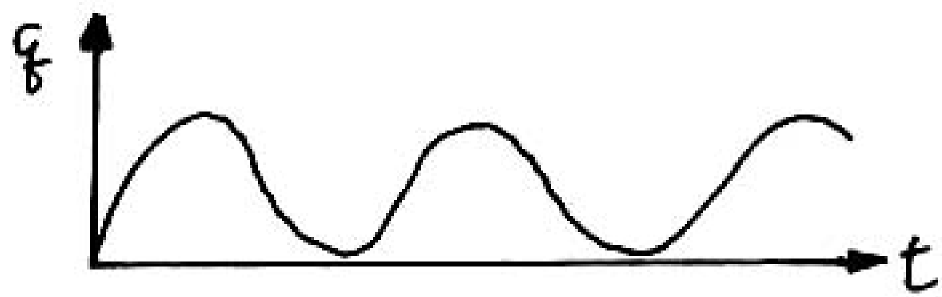
B) Push with an oscillating magnitude

Figure 19.4 **Representation of force with oscillating magnitude**



The response also oscillates

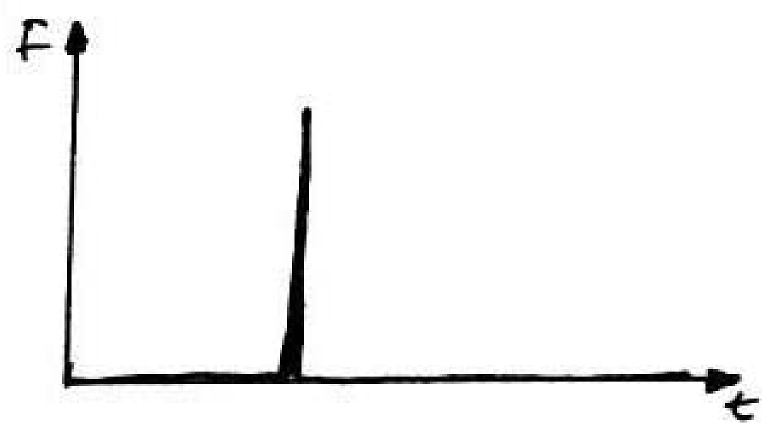
Figure 19.5 **Representation of oscillating response**



C) Whack mass with a hammer

⇒ Force is basically a unit impulse

Figure 19.6 **Representation of unit impulse force**

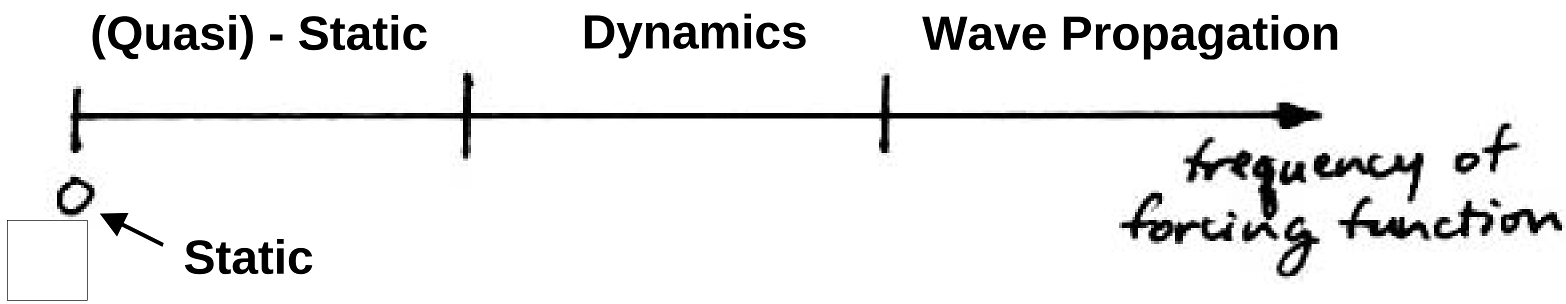


Force has very high frequencies

Response is (structural) waves in spring with no global deflection

--> Represent this as

Figure 19.7 **Representation of regions of structural response versus frequency of forcing function**



What determines division points between regimes?

--> borderline between quasi-static and dynamic is related to natural frequency of <u>structure</u>. Depends on:

- structural stiffness
- structural "characteristic length"

--> gives natural frequency of structure

--> borderline between dynamic and waves is related to speed of waves (sound) in <u>material</u>. Depends on:

- modulus
- density

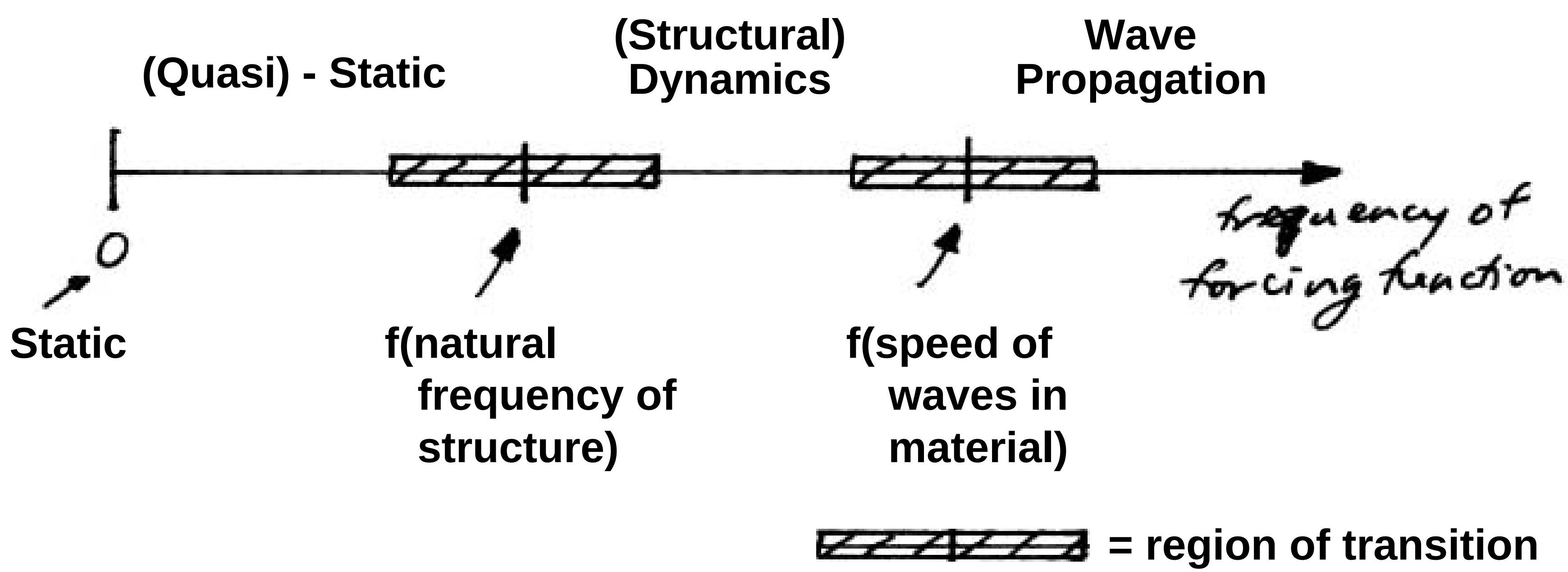
$$\text{speed} = \sqrt{E/\rho}$$

--> These are not well-defined borderlines

- depends on specifics of configuration
- actually transition regions, not borders
- interactions between behaviors

So illustration is:

Figure 19.8 **Representation of regions of structural response versus frequency of forcing function**



Statics -- Unified and 16.20 to date
Waves -- Unified
(Structural) Dynamics -- 16.221 (graduate course).

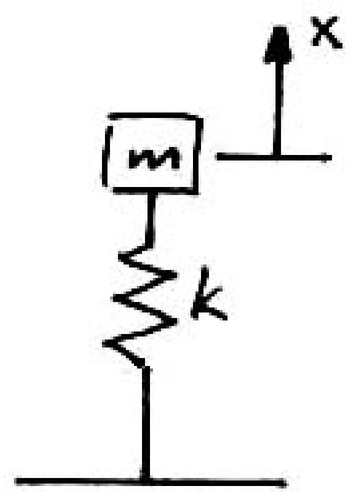
Look at what we must include/add to our static considerations

Consider the simplest ones…

The Spring-Mass System

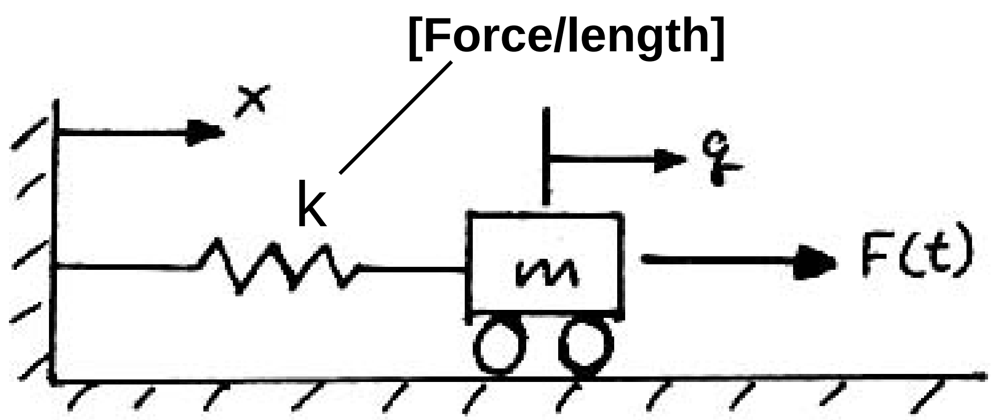
Are probably used to seeing it as:

Figure 19.9 **General representation of spring-mass system**



For easier relation to the structural configuration (which will later be made), draw this as a rolling cart of mass attached to a wall by a spring:

Figure 19.10 **Alternate representation of spring-mass system**



- The mass is subjected to some force which is a function of time
- The position of the mass is defined by the parameter q
- Both F and q are defined positive in the positive x-direction

  Static equation: F = kq
- What must be added in the dynamic case?

  Inertial load(s) = - mass x acceleration

In this case:

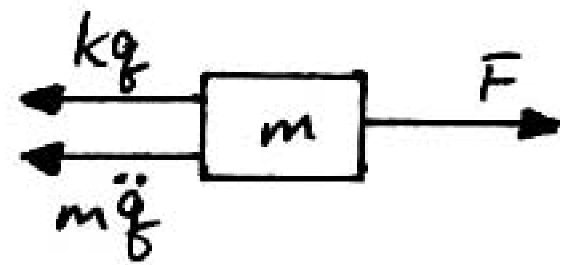
$$\text{inertial load} = -m\ddot{q}$$

where:

$$(\dot{\ }) = \frac{d}{dt} \quad \text{(derivative with respect to time)}$$

Drawing the free body diagram for this configuration:

Figure 19.11 **Free body diagram for spring-mass system**



$$\sum F = 0 \Rightarrow F - kq - m\ddot{q} = 0$$

$$\Rightarrow \boxed{m\ddot{q} + kq = F(t)}$$

← Basic spring-mass system (no damping)

This is a 2nd order Ordinary Differential Equation in <u>time</u>.

When the Ordinary/Partial Differential Equation is in <u>space</u>, need *<u>Boundary</u>* Conditions. Now that the Differential Equation is in time, need *<u>Initial</u>* Conditions.

2nd Order ⇒ need 2 Initial Conditions

Here:

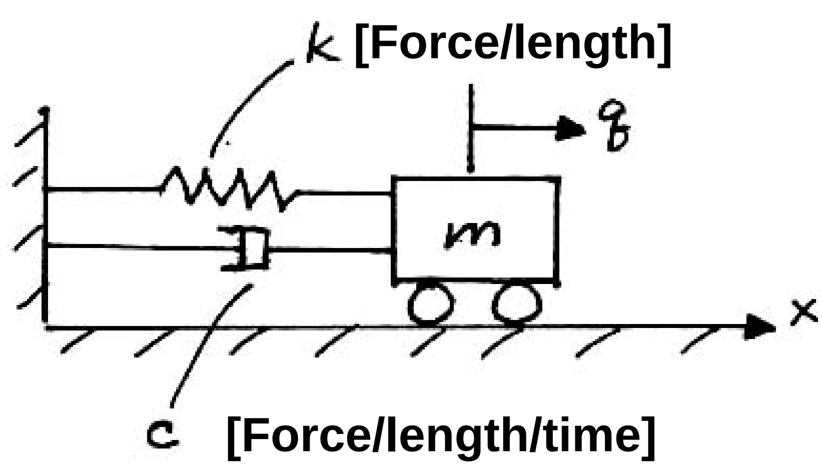
$$@ \quad t = 0 \quad \left.\begin{aligned} q &= q_0 \\ \dot{q} &= \dot{q}_0 \end{aligned}\right\} \text{ some initial values given (may often be zero)}$$

Will look at how to solve this in the next unit.

There is another consideration that generally occurs in real systems -- *DAMPING*.

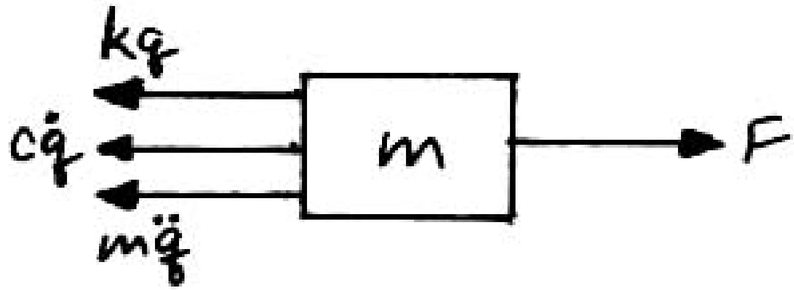
For the spring-mass system, this is represented by a dashpot with a constant c which produces a force in proportion to the velocity:

Figure 19.12 **Representation of spring-mass system with damping**



Here the free body diagram is:

Figure 19.13 **Free body diagram of spring-mass system with damping**



$$\sum F = 0$$

$$\Rightarrow \boxed{m\ddot{q} + c\dot{q} + kq = F(t)}$$

Basic spring-mass system
(with damping)

From here on: <u>neglect</u> <u>damping</u>

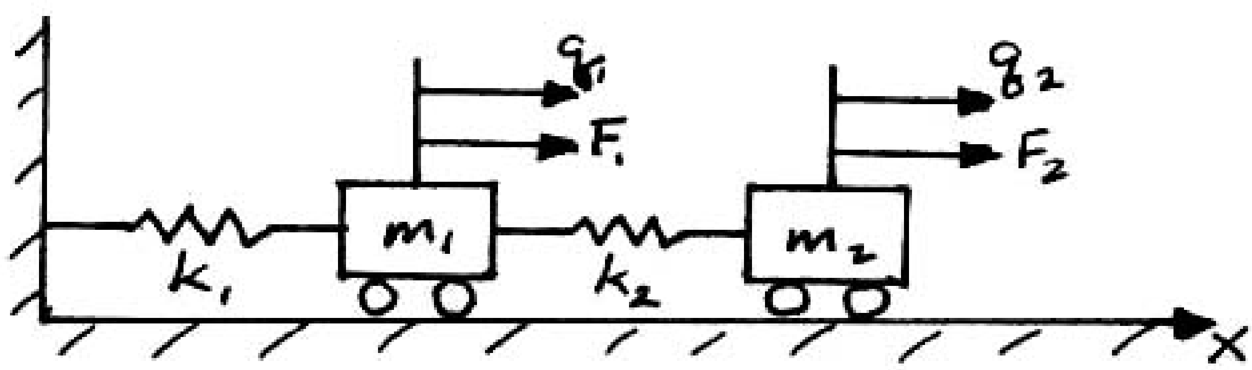
Can build on what has been done and go to a…

Multi-Mass System

For example, consider two masses linked by springs:

Each mass has stiffness, (k_i) mass (m_i) and force (F_i) with associated deflection, q_i

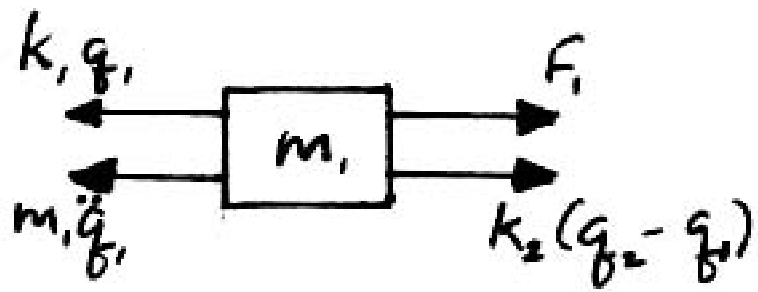
Figure 19.14 **Representation of multi-mass (and spring) system**



Consider the free body diagram for each mass:

- <u>Mass1</u>

Figure 19.15 **Face body diagram of Mass 1 in multi-mass system**



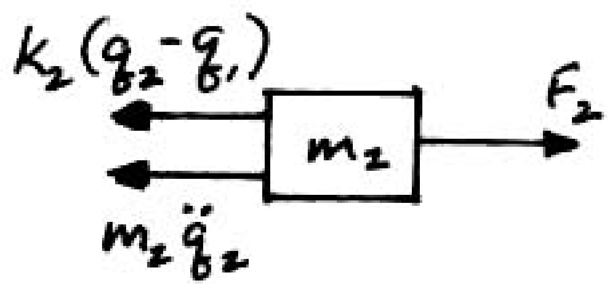
$$\sum F = 0$$

yields:

$$F_1 + k_2 \left(q_2 - q_1\right) - k_1 q_1 - m_1 \ddot{q}_1 = 0$$

- <u>Mass 2</u>

Figure 19.16 **Free body diagram of Mass 2 in multi-mass system**



$$\sum F = 0$$

yields:

$$F_2 - k_2\left(q_2 - q_1\right) - m_2\,\ddot{q}_2 = 0$$

Rearrange and unite these as (grouping terms):

$$m_1\,\ddot{q}_1 + \left(k_1 + k_2\right)q_1 - k_2\,q_2 = F_1$$

$$m_2\,\ddot{q}_2 - k_2\,q_1 + k_2\,q_2 = F_2$$

--> Two coupled Ordinary Differential Equations

Write in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

or:

$$\underset{\sim}{m}\,\underset{\sim}{\ddot{q}} + \underset{\sim}{k}\,\underset{\sim}{q} = \underset{\sim}{F}$$

↑ mass matrix ↑ stiffness matrix

Note that the stiffness matrix is symmetric (as it has been in all other considerations)

$$k_{ij} = k_{ji}$$

This formulation can then be extended to 3, 4….n masses with

m_i = mass of unit i

k_i = stiffness of spring of unit i

q_i = displacement of unit i

F_i = force acting on unit i

etc.

Will next consider solutions to this equation. But first talk about why these considerations are important in structures.

First issue -- what causes such response are:

Dynamic Structural Loads

Generic sources of dynamic loads:

- Wind (especially gusts)
- Impact
- Unsteady motion (inertial effects)
- Servo systems

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How are these manifested in particular types of structures?

Aircraft

- Gust loads and turbulence flutter

  (aeroelasticity is interaction of aerodynamic, elastic and inertial forces)

- Servo loads (and aero loads) on control surfaces

Spacecraft

Automobiles, Trains, etc.

Civil Structures

Earthquakes and Buildings

What does this all result in?

A response which is comprised of two parts:

- rigid-body motion
- elastic deformation and vibration of structure

Note that:

- Peak dynamic deflections and stresses can be several times that of the static values
- Dynamic response can (quickly) lead to fatigue failure
  (Helicopter = a fatigue machine!)
- Discomfort for passengers
  (think of a car without springs)

So there is a clear need to study structural dynamics

Before dealing with the continuous structural system, first go back to the simple spring-mass case and learn:

- Solutions for spring-mass systems
- How to model a continuous system as a discrete spring-mass system

then…

- Extend the concept to a continuous system