Practice Problems

5. For this problem, we will make use of the membrane analogy that was discussed in class and Unit #11 in the notes.

Specifically the membrane analogy for the torsion of narrow rectangular cross-sections will be used because each section of the channel cross-section and I cross-section beam can be represented as narrow rectangular cross-sections (see pp. 11~21 in unit #11).

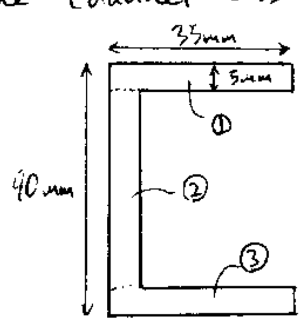
a). The angle of twist, α, can be obtained from

$$\frac{d\alpha}{dz} = \frac{T}{GJ}$$

$$\Rightarrow \alpha = \int_0^l \frac{T}{GJ} dz = \frac{Tl}{GJ} \quad \text{--- } ①$$

since T and J are constants.

So, we need to find the torsional constant J. Let's consider the channel cross-section first.

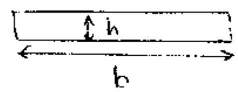


torsional constant of channel

$$J_c = J_① + J_② + J_③ \quad \text{--- } ②$$

From the membrane analogy of a narrow rectangular cross section, we know that J for a rectangle of length b and height h,

$$J = \frac{bh^3}{3}$$



Thus,

$$J_{①} = J_{③} = \frac{(35mm)(5mm)^3}{3} = 1.5\times10^{-9}\,m^4$$

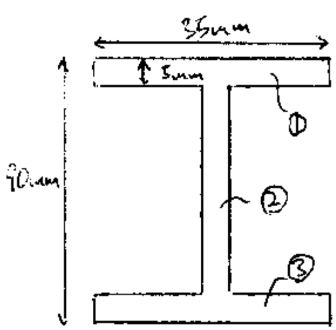
$$J_{②} = \frac{(80mm)(5mm)^3}{3} = 3.3\times10^{-9}\,m^4$$

Plugging the values for $J_{①}$, $J_{②}$ and $J_{③}$ in equation ②, we get

$$J_c = (1.5\times10^{-9}) + (3.3\times10^{-9}) + (1.5\times10^{-9})$$

$$\Rightarrow J_c = 6.3\times10^{-9}\,m^4$$

Next, let's consider the I-beam.



$$J_I = J_{①} + J_{②} + J_{③}$$

The I-beam is made from the same three pieces as the channel section, so the two sections must have the same torsional constants

$$J_I = J_c = 6.3 \times 10^{-9} m^4$$

To find the angle of twist, α, plug the given values into equation ①.

$$* \; G = \frac{E}{2(1+\nu)} = 27 \, GPa$$

For channel cross section :

$$\alpha_c = \frac{(40 Nm)(2m)}{(27 GPa)(6.3 \times 10^{-9} m^4)} = 0.47 \text{ rad.}$$

$$\therefore \alpha_c = 27°$$

Since the I-beam has the same torsional constant as the channel section, $\alpha_I = \alpha_c$

$$\Rightarrow \boxed{\alpha_I = 0.47 \text{ rad} = 27°}$$

b) The maximum resultant shear, τ_{max} is

$$\tau_{max} = \sqrt{\sigma_{yz}^2 + \sigma_{xz}^2} \quad \text{--- ③}$$

From the derivation of the torsion of a narrow rectangular cross-section, we found,

$$\sigma_{yz} = \frac{2T}{J} x$$

$$\sigma_{xz} = 0$$

$$\therefore \quad \tau_{max} = \frac{2T}{J} x_{max}$$

We will find τ_{max} in the widest part of the cross-section, where the local coordinate x is maximized. In the widest part,

$$x_{max} = \frac{1}{2} t$$

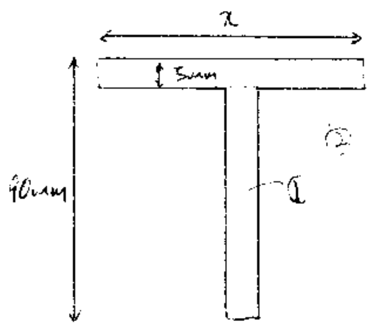
thickness

For both the channel and the I cross-section beams, the thickness, $t = 5mm$, so,

$$\tau_{max\,I} = \tau_{max\,C} = \frac{\cancel{2}T}{J}\frac{1}{\cancel{2}}t = \frac{(40Nm)}{(6.3\times10^{-9}m^4)}(0.005m)$$

$$\therefore \quad \boxed{\tau_{max\,I} = \tau_{max\,C} = 32\ MPa}$$

c) In order for the resistance to be the same, the torsional constant needs to be the same. The torsional constant for the T-section is



$$J = J_{①} + J_{②}$$

$$= \frac{(85\,mm)(5\,mm)^3}{3} + \frac{x(5\,mm)^3}{3} \quad \text{------} ④$$

Note that $J_c = J_I$ and

$$J_c = J_I = \frac{(80\,mm)(5\,mm)^3}{3} + \frac{2(35\,mm)(5\,mm)^3}{3} \quad \text{------} ⑤$$

Comparing equations ④ and ⑤, we get,

$$\frac{(85\,mm)\cancel{(5\,mm)^3}}{\cancel{3}} + \frac{x\cancel{(5\,mm)^3}}{\cancel{3}} = \frac{(80\,mm)\cancel{(5\,mm)^3}}{\cancel{3}} + \frac{2(35\,mm)\cancel{(5\,mm)^3}}{\cancel{3}}$$

$$\Rightarrow \quad x = (70\,mm) - (5\,mm)$$

$$\therefore \quad \boxed{x = 65\,mm}$$