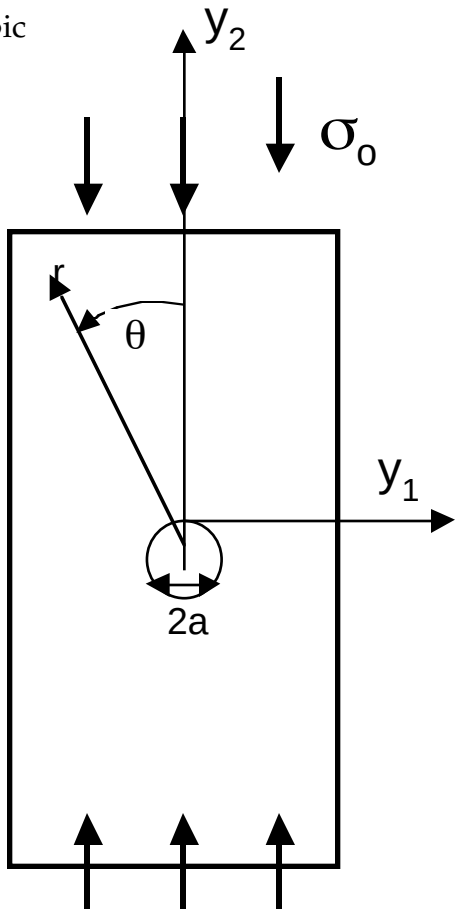
**16.20** **Handed Out: Lecture 16**
**Due: Lecture 20**

# HOME ASSIGNMENT #4

## Warm-Up Exercises

Let's consider the stress field around the hole in an isotropic plate loaded in uniaxial compression (no buckling occurs):




1. Derive expressions for the stress components at the hole boundary (r = a) as a function of θ in the $y_1$ - $y_2$ - $y_3$ coordinate system.

2. Plot these values, normalized by the applied far-field stress, $\sigma_o$ , as a function of θ.

3. Plot the "polar coordinate" stresses at the hole boundary as a function of θ.

4. Compare the plots of #2 and #3. Discuss any similarities/differences/noteworthy points.

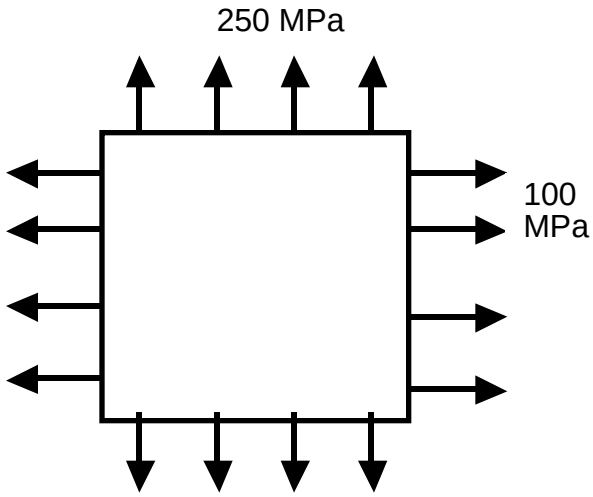
## Practice Problems

5. A pressurized aluminum vessel results in a state of plane stress in the skin of 100 MPa in one direction and 250 MPa perpendicular to this direction. Aluminum has a modulus of approximately 70 GPa and a Poissons's ratio of 0.30. Using the stress function approach:

(a) Construct a solution for this configuration.

(b) Determine the full strain state.

(c) Determine the entire deformation field.



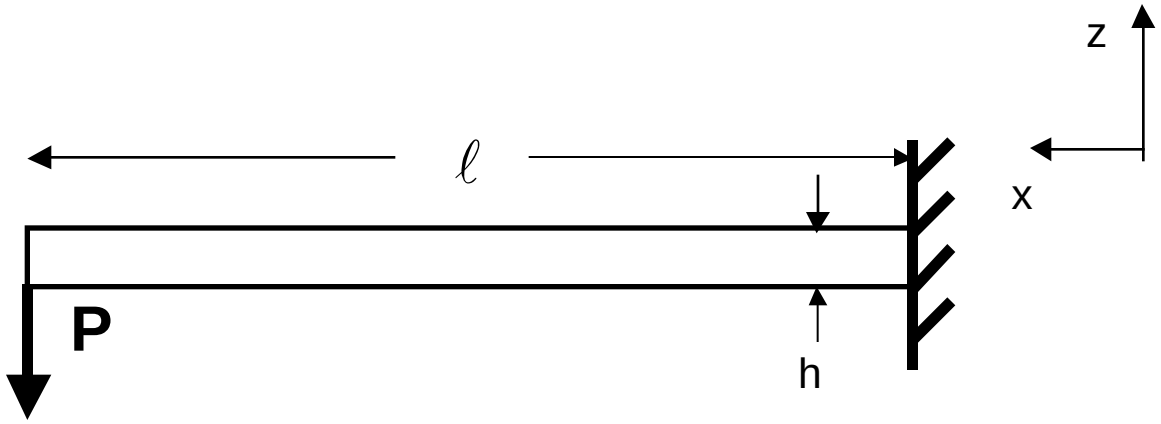

## Application Tasks

6. Use appropriate combinations of the Airy stress functions provided to:

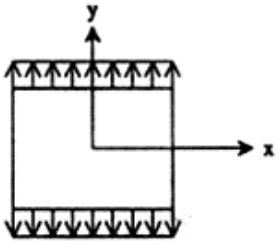
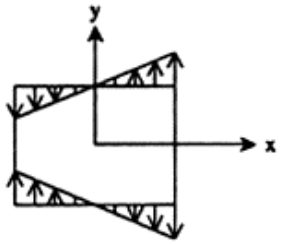
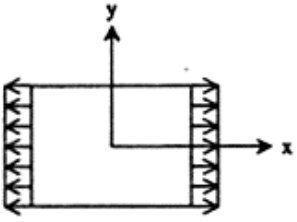
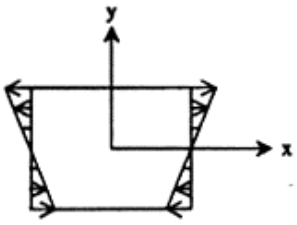
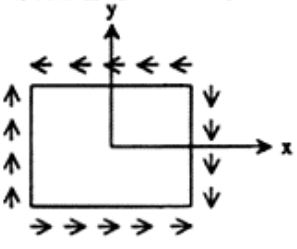
   (a) Construct a solution for the case of an isotropic cantilevered beam loaded by an end load as pictured.

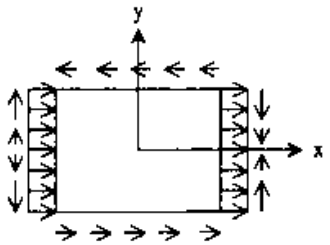
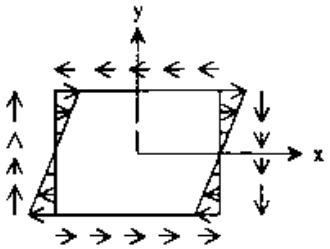
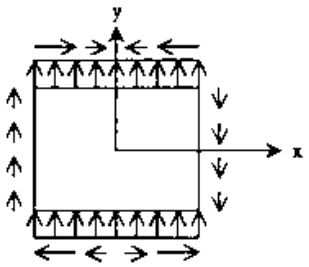
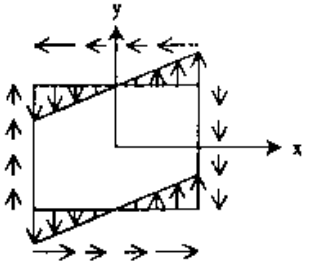
   (b) Compare these results (for $\sigma_{xx}$ and $\sigma_{xz}$) with those from simple beam theory.

   (c) Where in this problem is it necessary to apply St. Venant's principle?



$$\varphi = C_{ij}\, x^i\, y^j$$

| Stress function φ | $\sigma_{xx}$ | $\sigma_{yy}$ | $\sigma_{xy}$ | Surface Forces on Rectangle |
|---|---|---|---|---|
| $C_{20}\, x^2$ | 0 | $2\, C_{20}$ | 0 |   |
| $C_{30}\, x^3$ | 0 | $6\, C_{20}\, x$ | 0 |   |
| $C_{02}\, y^2$ | $2\, C_{02}$ | 0 | 0 |   |
| $C_{03}\, y^3$ | $6\, C_{03}\, y$ | 0 | 0 |   |
| $C_{11}\, x\, y$ | 0 | 0 | $-C_{11}$ |   |

| | | | | |
|---|---|---|---|---|
| $C_{12}\, x\, y^2$ | $2\, C_{12}\, x$ | 0 | $-2\, C_{12}\, y$ |  |
| $C_{13}\, x\, y^3$ | $6\, C_{13}\, x\, y$ | 0 | $-3\, C_{13}\, y^2$ |  |
| $C_{21}\, x^2\, y$ | 0 | $2\, C_{21}\, y$ | $-2\, C_{21}\, x$ |  |
| $C_{31}\, x^3 y$ | 0 | $6\, C_{31}\, x\, y$ | $-3\, C_{31}\, x^2$ |  |