16.13 Aerodynamics of Viscous Fluids Handed out: 22 Oct 03
Problem Set 5 Due: 5 Nov 03

Write a program to calculate quasi 1-D viscous flow in a diffuser using Interacting Boundary Layer Theory with the two-equation integral method described in class. The geometry to be used is sketched below. Only half the diffuser will be treated, assuming symmetry about the center plane, which can also be considered as an inviscid wall.
The geometry is defined by:

$$h(x) \;=\; h_0 \;+\; (h_1 - h_0)\,\left(1.5\,x^2 \;-\; \overset{0.5}{\cancel{0.6}}\,x^5\right) \qquad\qquad 0 \le x \le 1$$

One can choose the scales as $L = 1$, and $u_o = 1$ without loss of generality. The corresponding Reynolds number is $Re = u_0 L/\nu$.

Both laminar and turbulent boundary layers will be considered. The integral BL equations derived in class are valid for turbulent flows, but the empirical closure functions $H^*(H)$, $C_f(H, Re_\theta)$, etc., quantitatively differ between the turbulent and laminar case. The downloadable files contain these functions for both cases implemented as simple Fortran subroutines.
It is realistic to assume that the flow is laminar from the leading edge, and turbulent downstream of some transition location. We will assume that the transition location is where the shape parameter H first exceeds some critical value H_{crit}, and turbulent flow then persists downstream. This is a rather crude transition model, but it will adequately simulate transition induced by pressure gradients and/or laminar separation.

Your program needs to integrate the governing ODEs for $\theta(x)$, $\delta^*(x)$, $u_e(x)$ along the lines of the attached writeup. Your program should have the option to solve the Classical problem. Use the tabulated Blasius solution or Thwaites' method to generate θ, δ^*, and u_e at the first point where $x > 0$. This is necessary to start the downstream integration.

1a) Set the channel geometry parameters to $h_0 = 0.05$ and $h_1 = 0.07$. Calculate both Classical and IBLT solutions for $Re = 10^4, 10^5, 10^6$ with $H_{\text{crit}} = 500$ (fully laminar). Plot $u_e(x)$, $\theta(x)$, $H(x)$, and the geometry with the displacement surface superimposed (you might also want to compare the inviscid and viscous $u_e(x)$ distributions as a check). The Classical solution will encounter the
4 Goldstein singularity at a separation point and fail, whereas the IBLT solution should not. Explain this difference in behavior by considering for each case the solvability of the 3×3 system matrix near a separation point. What is the physical mechanism which makes the system solvable or insolvable?

1b) Compare the separation locations for the cases computed in 1). Why does the IBLT separation location depend on the Reynolds number while the Classical location does not?

2a) Set the channel geometry parameters to $h_0 = 0.05$ and $h_1 = 0.17$, and set $Re = 5 \times 10^5$.
3 Calculate IBLT solutions over the range $2.6 \le H_{\text{crit}} \le 15$. You should observe that the transition location moves steadily downstream as H_{crit} is increased, with a *separation bubble* forming when $H_{\text{crit}} > 4$. Plot the $u_e(x)$ curves for all H_{crit} values overlaid to discern the movement of the bubble. Do the same for $\theta(x)$ and $H(x)$.

3a) If the BL were growing along an airfoil surface, the trailing edge θ would indicate the drag. Calculate and plot $\theta(1)$ versus H_{crit} over the range $2.6 \leq H_{\text{crit}} \leq 10$, with $Re = 2 \times 10^5$. You should observe that the transition location moves steadily downstream as H_{crit} is increased, with a *separation bubble* forming when $H_{\text{crit}} > 4$. Plot the $u_e(x)$ curves for all H_{crit} values overlaid to discern the movement of the bubble. Do the same for $\theta(x)$ and $H(x)$.

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3b) As transition moves downstream, the extent of the relatively high turbulent skin friction decreases, and one would expect continually less and less friction drag and a continuously decreasing $\theta(1)$. Why then is there a minimum in $\theta(1)$? Explain in terms of the integral momentum equation

$$\frac{d\theta}{dx} = \ldots$$

Does there appear to be an "optimum" transition location? If so, is there a simple rule for determining this location?

A *Stradford pressure recovery* is an adverse $u_e(x)$ distribution which produces nearly-separated flow everywhere. This $u(x)$ therefore has the fastest decrease possible, so that the diffuser is as short as possible. Consequently, the friction-producing area is a minimum, and so the loss is supposedly at a minimum.

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4) Modify your ODE system to allow a "design" calculation of the diffuser shape $h(x)$ necessary to achieve a specified constant $H(x) = H_{\text{spec}}$ corresponding to nearly-separated flow. *Hint: For simplicity, augment your equations to a* 4×4 *system.*

The design requirement is $u_e(x_{\text{exit}})/u_o = 0.5$. Two possible design objectives are:

i) minimum length, i.e. minimum necessary x_{exit}

ii) minimum loss, i.e. minimum $\theta(x_{\text{exit}})$

Determine the best H_{spec} for each objective. If they are markedly different, explain.

Use $Re = 10^6$, and a constant $h(x) = h_0$ with laminar Blasius flow until $x = 0.1$. Force the flow turbulent at $x = 0.1$, and calculate $h(x)$ thereafter.

Bonus Question:

Similar to problem set 4, suction can be used to suppress separation in the diffuser. Assume that the flow is removed through a porous surface so that $U_w = 0$ is a reasonable assumption at the diffuser wall.

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5a) Derive the integral momentum and kinetic energy equations for a boundary layer subjected to suction along the wall. Is this valid for both laminar and turbulent flows ? If not, what changes would be necessary ? Modify the interaction law for the quasi-1D flow to account for the mass removed by suction. Implement the modified boundary layer equations and interaction law in your IBLT solver.

0.07

5b) Set $h_1 = \cancel{0.2}$ and $H_{crit} = 500$ for fully laminar flow, and test the code for different values of constant suction ($v_w(x) = const.$) distributed from $x = 0.1$ to $x = 1$. How much suction flow is required to suppress separation ? Plot the mixed out total pressure loss using the expression given below for different values of suction flow. Is there a minimum in the loss variation ? Note that you can integrate the suction distribution and express the total suction flow as a fraction of the inlet flow ($C_q = \dot{m}_{suction}/\dot{m}_{inlet}$).

5c) Modify your ODE system to allow a "design" calculation of the suction distribution $v_w(x)$ necessary to achieve a specified constant $H(x) = H_{\text{spec}}$ for a fixed diffuser geometry. Determine the H_{spec} that results in a minimum total pressure loss for the diffuser. Plot the calculated suction distribution, and calculate the total suction mass flow required as a fraction of the inlet mass flow.

Use $Re = 10^6$, and a constant $h(x) = h_0$ with laminar Blasius flow until $x = 0.1$. Force the flow turbulent at $x = 0.1$, and calculate ~~$h(x)$~~ thereafter.

The mixed-out total pressure loss coefficient at the exit of a diffuser is given by:

$$C_{pt} = \frac{P_{tm} - P_{t\infty}}{1/2\rho U_\infty^2} = \frac{2\theta}{h_1} + \left(\frac{\delta^*}{h_1}\right)^2$$

where h_1 is the height of the diffuser at the exit.

$h_1 = 0.2$ $\qquad$ $h_1 = 0.17$ $\qquad$ $h_1 = 0.07$

C_a	C_{pt}	
0	0.652	0.0653.

C_a	C_{pt}	H
0	0.0653.	
0.0018	0.0602	
0.007	0.04567	15
0.010	0.03950	5.6
0.014	0.03806	2.55
0.013	0.03860	3.3
0.018	0.03795	1.99
0.0216	0.03856	1.724
0.0252	0.0394	1.58