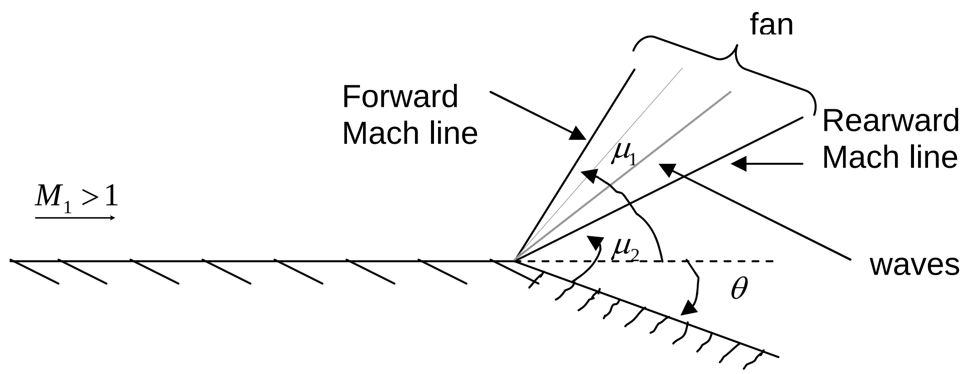
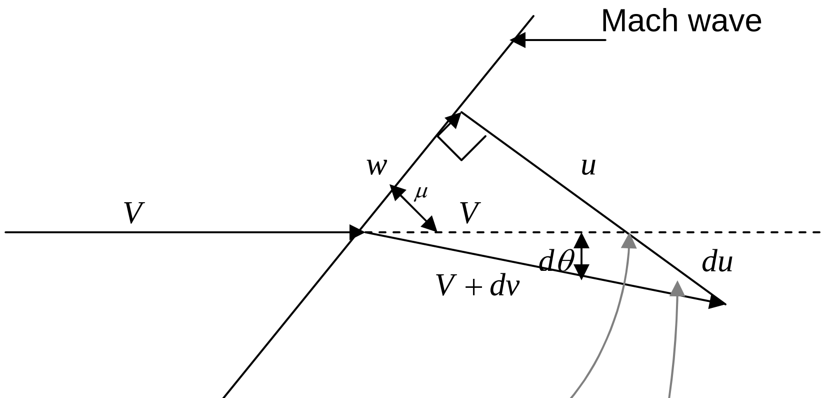
Prandtl-Meyer Expansion Waves

When a supersonic flow is turned around a corner, an expansion fan occurs producing a higher speed, lower pressure, etc. in an <u>isentropic</u> process.



∗ Just as we saw with shock waves, if we apply conservation of mass and momentum across a single wave, the tangential velocity is unchanged.
∗ Unlike a shock wave, an expansion wave is isentropic.

So let's pick out a single wave:



From law of sines:

$$\frac{V + dV}{V} = \frac{\sin\left(\frac{\pi}{2} + \mu\right)}{\sin\left(\frac{\pi}{2} - \mu - d\theta\right)}$$

Using $d\theta \to 0 \& \sin\mu = \frac{1}{M}$, we can find :

$$\boxed{d\theta = \sqrt{M^2 - 1}\,\frac{dV}{V}}$$

Next, $v = M_a \Rightarrow \frac{dv}{v} = \frac{dM}{M} + \frac{da}{a}$

Using adiabatic relationships, we can re-write:

$$\frac{\sqrt{\gamma R T_o}}{\sqrt{\gamma R T}} = \frac{a_o}{a} = \sqrt{\frac{T_c}{T}} = 1 + \frac{\partial - 1}{\partial} M^2$$

$$\Rightarrow \frac{da}{a} = -\left(\frac{\gamma - 1}{2}\right) M \left(1 + \frac{\gamma - 1}{\gamma} M^2\right)^{-1} dM$$

$$\Rightarrow \frac{dv}{v} = \frac{1}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M} \Rightarrow \boxed{d\theta = \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M}}$$

Finally, integrating $d\theta$ we find:

$$\theta = v(M_2) - v(M_1)$$

Where

$$v(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma - 1}(M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

Prandtl-Meyer function

Problem: Estimate the rates of the pressure inside the pitot probe to the freestream static pressure, p_a / p_∞ .