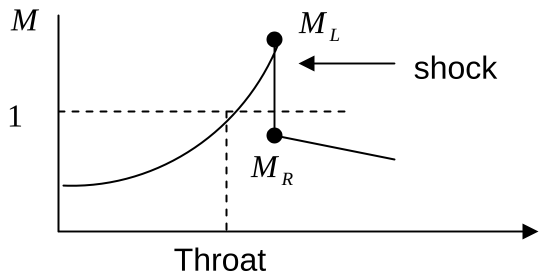
Normal Shock Waves

In our quasi-1D flows, shocks can occur from a supersonic-to-subsonic state. These shocks are discontinuous in our inviscid flow model (recall that shocks are very thin and their thickness scales with $1/R_e$):



Upstream Mach: $M_L > 1$
Downstream Mach: $M_R < 1$

The shock jump relationships come from the conservation equations we have seen before:

$$\rho_L U_L A_L = \rho_R U_R A_R$$
$$(\rho_L u_L^2 + p_L) A_L = (\rho_R u_R^2 + p_R) A_R - \int_{A_L}^{A_R} p dA$$
$$\rho_L u_L h_{o_L} A_L = \rho_R u_R h_{o_R} A_R$$

Since the jump is discontinuous (i.e. it has zero thickness)

$A_L = A_R = A \quad \Rightarrow dA = 0$

$$\Rightarrow \quad \boxed{\begin{aligned} \rho_L u_L &= \rho_R u_R \\ \rho_L u_L^2 + p_L &= \rho_R u_R^2 + p_R \\ \rho_L u_L h_{0_L} &= \rho_R u_R h_{o_R} \end{aligned}}$$

Ranhine-Hugoniot Shock Jump Relationships

Here are some important things to know about shock waves from these relationships:

* Mathematically, "shocks" exist which jump from subsonic-to-supersonic flow. However, these "shocks" can be shown to violate the 2nd Law ($\Delta s < 0$).
* Only shocks which jump from supersonic-to-subsonic states satisfy the 2nd Law. The Mach number downstream of shocks is given by :

$$M_R^2 = \frac{1 + \frac{1}{2}(\gamma - 1) M_L^2}{\gamma M_L^2 - \frac{1}{2}(\gamma - 1)} \quad \text{where } M_L > 1$$

And it can be shown that $\Delta s > 0$.

* The stagnation enthalpy (and therefore the total temperature) is constant through a shock (shocks are adiabatic).

$\Rightarrow$ $h_{0_L} = h_{0_R}$ or, equivalently,

$$T_{0_L} = T_{0_R}$$

* Total pressure decreases through a shock (this is a direct result of the entropy Increasing while $T_0 = const.$):

$$\Rightarrow \boxed{\frac{p_{0_R}}{p_{0_L}} = e^{-(s_R - s_L)/R}}$$