Falkner-Skan Flows

For the family of flows, we assume that the edge velocity, $u_e(x)$ is of the following form:

$u_e(x) = Kx^m$ $\qquad\qquad$ $K =$ arbitrary constant

The pressure can be calculated from the Bernoulli in the outer, inviscid flow:

$$p_e + \frac{1}{2}\rho u_e^2 = const.$$

$$\Rightarrow \frac{dp_e}{dx} = -\rho u_e \frac{du_e}{dx}$$

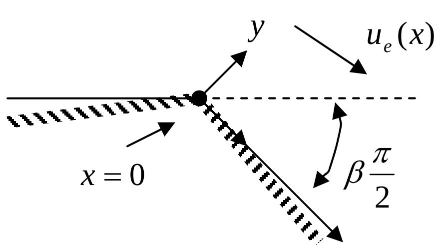
$$\Rightarrow \frac{dp_e}{dx} = -\rho K_m^2 x^{2m-1}$$

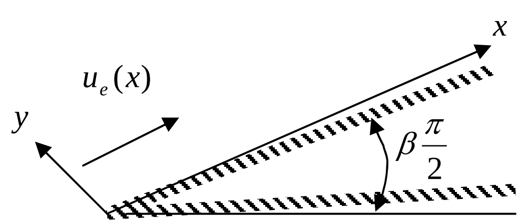
if $m > 0$ then $\frac{dp_e}{dx} < 0 \Rightarrow$ favorable pressure gradient

if $m < 0$ then $\frac{dp_e}{dx} > 0 \Rightarrow$ adverse pressure gradient

These edge velocities result from the following inviscid flows:

$$\beta \equiv \frac{2m}{1+m}$$





Flow around a corner (diffusion)
$-2 \le \beta \le 0$

Wedge flow
$0 \le \beta \le 2$

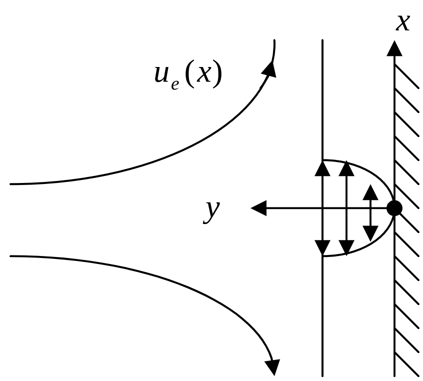
Some important cases:

$\beta = 0, m = 0$: flat plat (Blasius)
$\beta = 1, m = 1$: plane stagnant point

The boundary layer independent variable η from the Blasius solution generalizes to:

$$\eta \equiv y\sqrt{\frac{m+1}{2}\frac{u_e(x)}{\nu x}} \text{ and } u(x,y) = u_e(x) f^1(m)$$

An interesting case in $\beta = 1, m = 1$, i.e. stagnation point flow:



$\underbrace{u_e = K_x}$
inviscid flow velocity increases away from stag. pt. at $x = 0$

$$\eta = y\sqrt{\frac{1+1}{2}\frac{K_x}{\nu_x}}$$

$$\Rightarrow \qquad \eta = y\sqrt{\frac{K}{\nu}}$$

$\Rightarrow \eta$ is independent of x

$\Rightarrow$ Boundary layer at a stagnation point does not grow with x!

The skin friction can be found from:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_e(x) \underbrace{\left.\frac{d^2 f}{d\eta}\right|_{\eta=0}}_{f^{11}(o)} \left.\frac{\partial \eta}{\partial y}\right|_{y=0}$$

Since $\eta = y\sqrt{\frac{m+1}{2}\frac{u_e(x)}{vx}} \Rightarrow \frac{\partial \eta}{\partial y} = \sqrt{\frac{m+1}{2}\frac{u_e(x)}{vx}}$

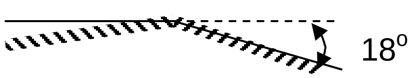
$$\Rightarrow \qquad \tau_w = \mu u_e(x)\sqrt{\frac{m+1}{2}\frac{u_e(x)}{vx}}\, f^{11}(o) \longleftarrow \text{tabulated}$$

The skin friction coefficient is normalized by $\frac{1}{2}\rho u_e^2(x)$:

$$C_f(x) \equiv \frac{\tau_w}{\frac{1}{2}\rho u_e^2(x)} = 2\sqrt{\frac{m+1}{2}\frac{v}{u_e(x)x}}\, f^{11}(o)$$

$$\Rightarrow \qquad \boxed{\begin{aligned} C_f &= \frac{2\sqrt{\frac{m+1}{2}}\, f^{11}(o)}{\sqrt{\mathrm{Re}_x}} \\ \mathrm{Re}_x &\equiv \frac{u_e(x)x}{v} \end{aligned}}$$

Note: separation occurs when $C_f = 0$ which means $f^{11}(o) = 0$. From the table, this occurs for $\beta = -0.19884$



$\Rightarrow$ This is only an angle of 18°!