Correlation Methods for Integral Boundary Layers

We will look at one particularly well-known and easy method due to Thwaites in 1949.

First, start by slightly re-writing the integral b.l. equation. We had:

$$\frac{\tau_w}{\rho_e u_e^2} = \frac{d\theta}{dx} + (2+H)\frac{\theta}{u_e}\frac{du_e}{dx}$$

Multiply by $u_e\theta / \nu$:

$$\frac{\tau_w \theta}{\mu u_e} = \frac{u_e\theta}{\nu}\frac{d\theta}{dx} + \frac{\theta^2}{\nu}\frac{du_e}{dx}(2+H)$$

Then define $\lambda = \frac{\theta^2}{\nu}\frac{du_e}{dx}$ and this equation gives:

$$u_e \frac{d}{dx}\left(\frac{\lambda}{du_e/dx}\right) = 2\left[\frac{\tau_w\theta}{\mu u_e} - \lambda(2+H)\right]$$

Thwaites then assumes a correlation exists which only depends on λ. Specifically:

$$H = H(\lambda) \quad \text{and} \quad \frac{\tau_w\theta}{\mu u_e} = S(\lambda)$$

$H(\lambda)$: shape factor correlation

$S(\lambda)$: shear correlation

$$\Rightarrow u_e \frac{d}{dx}\left(\frac{\lambda}{du_e/dx}\right) \cong 2\left[S(\lambda) - \lambda(2+H(\lambda))\right]$$

$\cong$: now this is an approximation

In a stroke of genius and/or luck, Thwaites looked at data from experiments and known analytic solutions and found that

$$u_e \frac{d}{dx}\left(\frac{x}{du_e/dx}\right) \approx 0.45 - 6\lambda \quad !!$$

This can actually be integrated to find:

$$\boxed{\theta^2 = \frac{0.45\nu}{u_e^{\,6}} \int_o^x u_e^{\,5} dx}$$

where we have assumed $\theta(x=0)=0$ for this.