16.100 Take-Home Exam #2
Fall 2005
Due: Friday, December 9, 2005 at 9am

Directions: This take-home exam is to be completed without help from any other individual except Prof. Darmofal. This includes help from other students (whether or not they are taking 16.100), the TA's, other faculty, people on the Internet, etc. Questions may be asked to Prof. Darmofal. In most cases, Prof. Darmofal's responses will be sent to the entire class.
You are permitted to use your books, class notes, library resources, etc. However, any resources you use in solving the problems should be cited. The ONLY resources you do not need to include are the Anderson textbook and any notes of mine (including those you might have from in-class). Any other books, web sites, etc. that you might have used MUST be cited.

After the written exams have been turned in and before you take your oral exam, you may discuss the written exam or other 16.100 content to help in preparing for the oral exam. This discussion can be with other class members as well as with the TA. **You may not discuss any 16.100 material once you have taken the oral exam (until after final exam week is over).**

Please sign this cover sheet, and turn it in with your solution.

I understand and have followed the directions for this take-home exam. The solutions I have turned in represent my understanding.

Signed:____

Date:____

Grade breakdown:

Problem #1 (Out of 30%)____________

Problem #2 (Out of 50%)____________

Problem #3 (Out of 20%)____________

TOTAL (Out of 100%)______________

Problem 1

In this problem, assume that the flow is inviscid and two-dimensional.

a) Determine the lift and drag coefficient for a flat plate at $M_\infty = 4$, $\alpha_\infty = 4$ degrees.

b) Determine the lift and drag coefficient for a flat plate at $M_\infty = 4$, $\alpha_\infty = 8$ degrees.

c) Using the previous results, estimate the lift slope (i.e. $dc_l / d\alpha_\infty$).

d) Using the previous results, show that the drag coefficient is well-approximated by a purely quadratic function of the angle of attack, i.e.

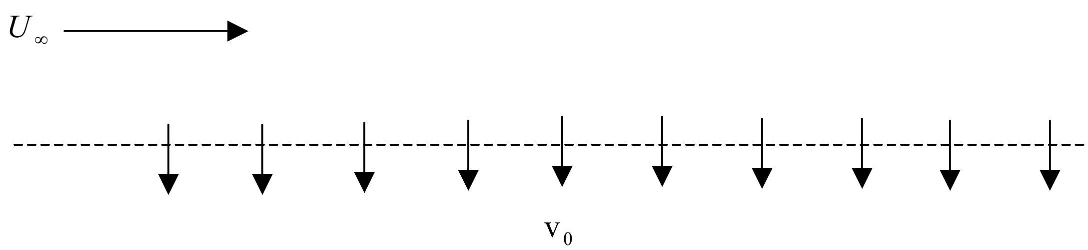
$$c_d = K\alpha_\infty^2$$

Specifically, determine K.

e) Determine the location of the aerodynamic center at small angles of attack for a flat plate at $M_\infty = 4$.

Problem 2

In this problem, we will consider the use of wall suction (i.e. removing flow in the boundary layer through a porous wall) to help control boundary growth as shown in the figure below where the freestream velocity is U_∞ and the suction velocity through the wall is v_0.

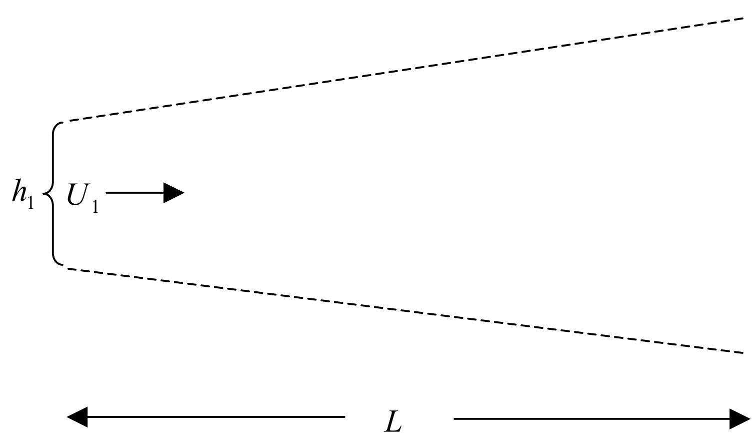


a) Show that the velocity field:

$$u(x,y) = U_\infty\left[1-\exp\left(-\frac{y\mathrm{v}_0}{\mu/\rho}\right)\right] \qquad \mathrm{v}(x,y) = -\mathrm{v}_0$$

is a solution to the incompressible, two-dimensional continuity and Navier-Stokes equations and satisfies the boundary conditions: $u(x,0)=0$, $u(x, y\to\infty) = U_\infty$, and $\mathrm{v}(x,0) = -\mathrm{v}_0$.

In the remaining parts of this question, you will analyze how much suction is required to control a boundary layer in the straight-walled diffuser shown below:



b) The diffuser is designed to decrease the velocity from $u_e(0) = U_1$ to $u_e(L) = \frac{1}{2}U_1$. Assuming that the x-velocity (outside of the boundary layers) is independent of y and that the boundary layers are thin, determine the edge velocity as a function of x, i.e. find $u_e(x)$ in the diffuser.

c) The integral boundary layer equation can be modified to include the wall suction velocity and is given by:

$$\frac{\tau_w}{\rho u_e^2} = \frac{\mathrm{v}_0}{u_e} + \frac{d\theta}{dx} + (2+H)\frac{\theta}{u_e}\frac{du_e}{dx}$$

Based on the analytic solution derived in part a), assume that the boundary layer velocity profile has the form,

$$u(x,y) = u_e(x)\left[1-\exp\left(-\frac{y}{\delta(x)}\right)\right]$$

and calculate τ_w, θ, and H. Then, substitute these results into the integral boundary layer equation. For the diffuser described above, determine the suction velocity distribution, $\mathrm{v}_0(x)$ required for the boundary layer thickness to remain constant, i.e. $\delta(x) = \text{constant}$.

d) For the following conditions:

$$\frac{\rho U_1 h_1}{\mu} = 10{,}000 \qquad\qquad \frac{\delta}{\mathrm{h}_1} = 0.05$$

Determine the ratio of the diffuser inlet height to length, h_1 / L, such that the maximum suction velocity is 1% of U_1. Plot $\mathrm{v}_0(x)/U_1$ for this situation.

Problem 3

Consider a flat plate at zero angle of attack in which the freestream conditions are:

$U_\infty = 100\ \text{m/s} \qquad p_\infty = 1.0\times10^5\ \ \text{N/m}^2 \qquad \rho_\infty = 1.2\ \ \text{kg/m}^3 \qquad \mu_\infty = 1.8\times10^{-5}\ \ \text{kg/(m s)}$

a) If the boundary layer becomes fully turbulent at $\text{Re}_x = 500{,}000$, determine the distance from the leading edge of the flat plate at which transition occurs.

b) At the transition location, assume that the boundary layer immediately (i.e. discontinuously) switches from laminar to turbulent. At the transition location, estimate the laminar boundary layer thickness and the turbulent boundary layer. Use Equations 18.23 and 19.1 from Anderson for these estimates.

c) The velocity profile in a laminar boundary layer on a flat plate is well approximated by $u/U_\infty = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$. The velocity profile in a turbulent boundary layer on a flat plate is well approximated by $u/U_\infty = (y/\delta)^{1/7}$. In both the laminar and turbulent cases, $u/U_\infty = 1$ for $y > \delta$. On the same graph, plot the laminar and turbulent profiles at the transition location as a function of u versus y.

d) For the turbulent boundary layer at the transition location, plot the following non-dimensional total pressure coefficient versus y:

$$C_{P_{tot}} \equiv \frac{P_{tot} - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}.$$