

**16.100 Take-Home Exam #1**  
Distributed: October 17 at 10am  
Due: October 24 at 9am

**Directions:** This take-home exam is to be completed without help from any other individual except Prof. Darmofal. This includes help from other students (whether or not they are taking 16.100), the TA's, other faculty, people on the Internet, etc. Questions may be asked to Prof. Darmofal. Prof. Darmofal's responses will be sent to the entire class. You are permitted to use your books, class notes, library resources, etc. However, any resources you use in solving the problems must be cited. The ONLY resources you do not need to include are the Anderson textbook and any notes of mine (including those you might have from in-class). Any other books, web sites, etc. that you have used MUST be cited.

The exam is due in class at 9am on October 24. Absolutely no late exams will be accepted.

Also, the content of the written and oral exams cannot be discussed until after the last oral exam (which will be Friday, October 28 at 5pm).

Please sign this cover sheet, and turn it in with your solution.

I understand and have followed the directions for this take-home exam. The solutions I have turned in represent my understanding.

Signed: \_\_\_\_\_

Date: \_\_\_\_\_

**Grade breakdown:**

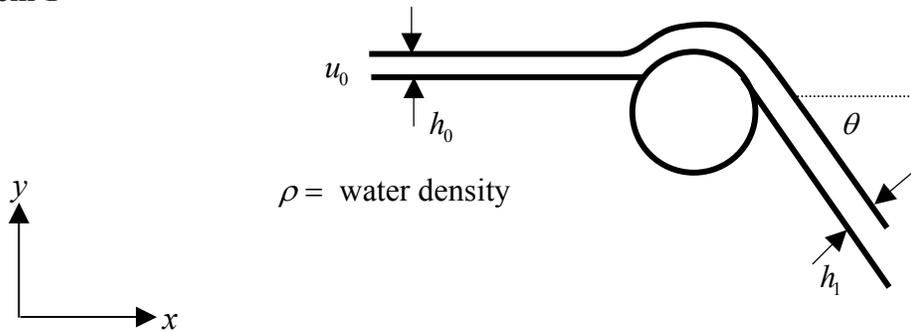
**Problem #1 (Out of 20%)** \_\_\_\_\_

**Problem #2 (Out of 30%)** \_\_\_\_\_

**Problem #3 (Out of 50%)** \_\_\_\_\_

**TOTAL (Out of 100%)** \_\_\_\_\_

### Problem 1



Consider a stream of water with initial velocity  $u_0$  and width  $h_0$  which impinges on a cylinder of radius  $R$  as shown in the figure. The stream eventually separates from the cylinder at an angle  $\theta$  and width  $h_1$ . Using the integral conservation equations, calculate the force in the  $x$  and  $y$  directions of the water acting on the cylinder? Assume the cylinder is fixed.

### Problem 2

The goal of this problem is to design the camberline of an airfoil using thin airfoil theory such that the following three constraints are met:

- The lift coefficient is 0.5.
- There is no suction peak at the leading edge (i.e. at  $x = 0$ )
- The airfoil has neutral static stability (i.e. the center of pressure and aerodynamic center are collocated).

A. Based on the three constraints, determine  $A_0$ ,  $A_1$ , and  $A_2$  (from thin airfoil theory). Also, determine the moment coefficient at the aerodynamic center. NOTE: you do not need any geometry information to do this!

B. Next, assume that the camberline is given by,

$$\frac{z}{c} = a_1 \frac{x}{c} \left(1 - \frac{x}{c}\right) + a_2 \frac{x}{c} \left(1 - \frac{x}{c}\right) \left(1 - 2\frac{x}{c}\right)$$

where  $a_1$  and  $a_2$  are constants to be determined. Find the values of  $\alpha$ ,  $a_1$ , and  $a_2$  such that the three design goals given above are met. Plot the camber line.

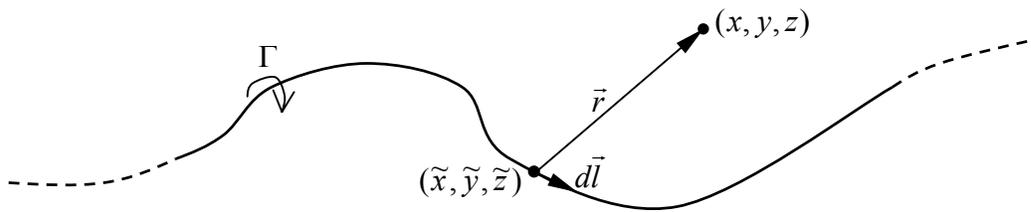
C. Plot the difference in pressure coefficient,  $\Delta C_p \equiv C_p|_{lower} - C_p|_{upper}$ .

### Problem 3

The following questions deal with vortex filaments and their use in modeling the flow on a wing. Recall that the velocity at  $(x, y, z)$  induced by a vortex filament of strength  $\Gamma$  can be found from the Biot-Savart law,

$$\vec{V}(x, y, z) = \frac{\Gamma}{4\pi} \int_{-\infty}^{+\infty} \frac{d\vec{l} \times \vec{r}}{r^3}$$

where  $\vec{r} = (x - \tilde{x})\vec{i} + (y - \tilde{y})\vec{j} + (z - \tilde{z})\vec{k}$ ,  $r = |\vec{r}|$ , and  $(\tilde{x}, \tilde{y}, \tilde{z})$  is a point on the filament as shown in the sketch below:



- Prove that the velocity field induced by the filament satisfies the incompressible conservation of mass at any point  $(x, y, z)$ .
- Prove that the velocity field induced by the filament is irrotational at any point  $(x, y, z)$  that is not on the filament (i.e. only consider points not on the filament).
- Consider an uncambered, rectangular wing with span  $b$  and chord  $c$ . Model this wing using a single horseshoe vortex with its bound vortex at the quarter-chord line ( $x = c/4$ ) of the wing as shown in figure below. Calculate the induced velocity of the horseshoe vortex at the control point which is located at  $(\frac{3}{4}c, 0, 0)$ .
- Continuing with the flow model from part C, determine the strength of the filament such that flow tangency is satisfied at the control point for an angle of attack,  $\alpha_\infty$ . Assume that the thickness of the wing and the angle of attack are small.
- The lift on the wing can be found using a three-dimensional form of the Kutta-Joukowski theorem, specifically,  $L = \rho V_\infty \Gamma b$ . Combining this with the results from part D, find the lift slope (i.e.  $dC_L / d\alpha_\infty$ ) for a wing with  $AR = 10$ .

