NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Final Exam

Date: December 17, 2007

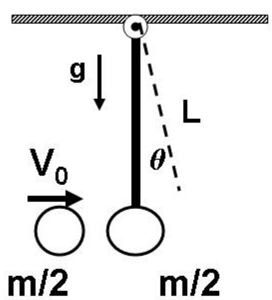
	Points
Problem 1	25
Problem 2	15
Problem 3	30
Problem 4	15
Problem 5	15

Problem 1 (25 points)

1.1) Consider a pendulum of mass $m/2$ at rest suspended by a massless rod of length L from a frictionless pivot in the presence of gravity, shown in a). Use small angle approximation for the subsequent motion of the pendulum, $\theta << 1$.

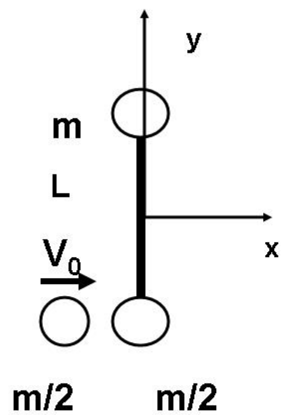
a) A mass $m/2$ with velocity V_0 collides **inelastically** (In an inelastic collision, the bodies do not rebound, but stick together.). What is the velocity of the two masses immediately after the collision? What is the instantaneous reaction force in the pivot? Show that these initial conditions follow from the conservation of linear and angular momentum. Is energy conserved in the collision?

b) What is the subsequent motion of the pendulum?



a)

1.2) Now consider a dumbbell at rest on a horizontal frictionless plane, as shown in b). It has 2 masses connected by a massless rigid rod; one of m, one of $m/2$. Another mass of $m/2$ and velocity V_0 impacts the lower mass and an **inelastic** collision occurs.



b)

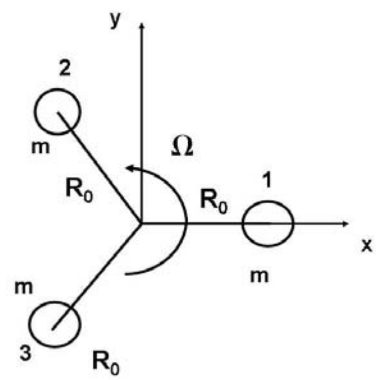
a) What is the initial velocity of the two dumbbell masses immediately after the collision? Show that these initial conditions follow from the conservation of linear and angular momentum.

b) Is the initial motion the same as that of 1.1?

c) What is the position of the center of mass with time? Is momentum conserved with time? Is angular momentum conserved with time? What is the force in the rod?

d) Solve for the position of the masses as a function of time in a coordinate system moving with the center of mass.

1.3) Now consider a three-mass dumbbell with the masses attached to three rigid massless rods of length R_0, **evenly and rigidly spaced in angle** as shown, spinning with angular velocity Ω. At $t = 0$ when the masses are in the positions shown, the linkage with mass 1 instantaneously breaks/disintegrates/dissapears.



a)What happens next? Qualitatively describe the resulting motion in the x,y coordinate system shown located at the initial position of the center of mass.

b) where is the center of mass for all time?

c) Is momentum conserved? Write an expression for the momentum.

d) Write an expression for the angular momentum. Is angular momentum about the center of mass conserved?

e) Is energy conserved?

f) Solve for the position of all three masses as a function of time.

Problem 2 (15 Points)

Due to various on-orbit disturbances, the shuttle orbit departs from the desired circular orbit and assumes the shape shown in the figure. Given the desired orbit radius R_o, and the parameters of the deformed elliptical orbit r_π and r_α, there two ways to recircularize the orbit using the approach of Hohmann transfer. Assume the orbits are co-planar.

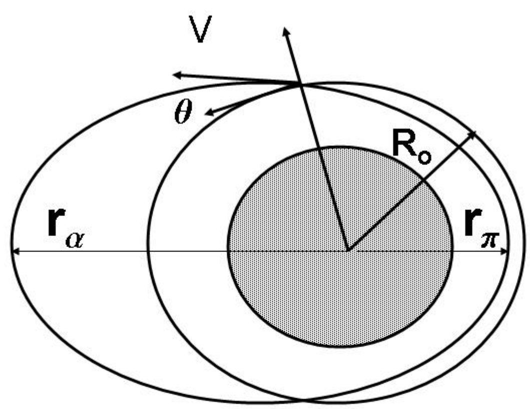
useful formulae

For an elliptic orbit, the velocity at perigee and apogee are

$$v_\pi^2 = \mu * \left(\frac{2}{r_\pi} - \frac{2}{(r_\pi + r_\alpha)}\right); v_\alpha^2 = \mu * \left(\frac{2}{r_\alpha} - \frac{2}{r_\pi + r_\alpha}\right). \quad (1)$$

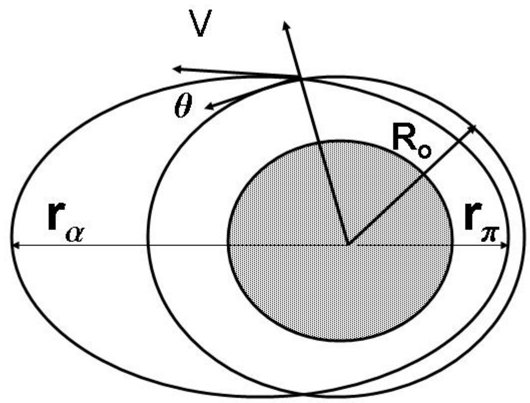
where r_π is the distance from the focus to perigee; r_α is the distance from the focus to the apogee.

a) the first method is to begin the orbit change at the current perigee. Describe the procedure and determine the total ΔV required for this method.



b) Sketch the various orbits used during this procedure.

c) The second method is to initiate the burn at the current apogee. Describe the procedure and determine the total ΔV required for this method.

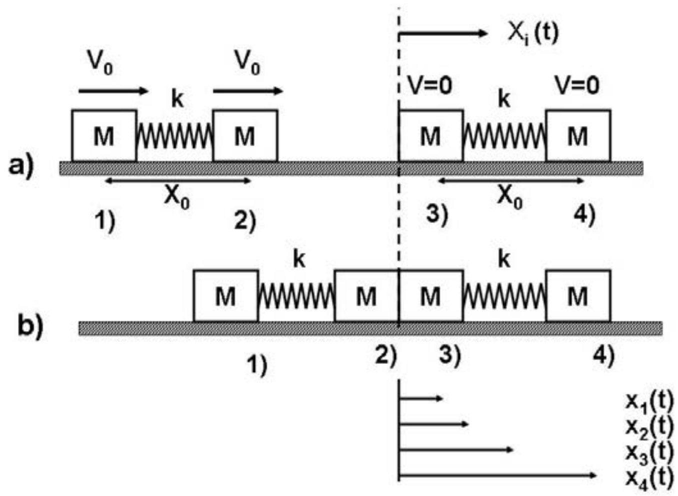


c)Sketch the various orbits used during this procedure.

d) Another possibility to recircularize the orbit would be to affect a change of both velocity and direction with a single ΔV where the orbits cross. Given θ and V, the velocity and inclination of the orbit as shown in the figure, what ΔV would be required?

Problem 3 (30 points)

Consider 4 blocks of mass M resting or sliding on a frictionless plane. Each pair of masses is separated by a spring of stiffness k. At the initial time, the springs are uncompressed, providing no force to the masses; the initial separation of the masses in each mass pair is X_0. Two of the masses are traveling at equal velocities V_0 as shown. Although the masses are shown with finite dimension, they are simply mass points. Therefore take their width as zero and their positions as the positions of their center of mass, x_1, x_2, x_3, x_4. Take the origin of coordinates at the initial impact point between $mass_2$ and $mass_3$.



3.1) At $t = 0$ an ***elastic*** collision occurs between $mass_2$ and $mass_3$ as shown.

3.1-a) Referring to the figure, what initial conditions for the configuration in figure b) are set by the collision i.e immediately after the collision? What is the velocity v_1 of $mass_1$ at t=0, the velocity v_2 of $mass_2$ at t=0, the velocity v_3 of $mass_3$ at t=0, and the velocity v_4 of $mass_4$ at t=0?

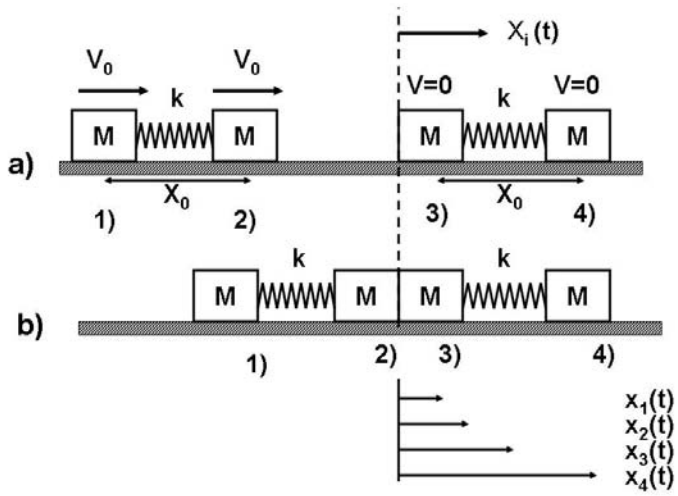
3.1) (CONTINUED)

3.1-b) What differential equations govern the motion of the masses? Write them. How many DE are there? What are the initial conditions?

3.1-c) Solve these equations for the position of the masses $x_1(t), x_2(t), x_3(t), x_4(t)$.

3.1-d) Describe the motion qualitatively. What is the position of the center of mass of the 4 mass system with time? Do the masses recontact after the initial collision? How do the masses respond?

3.2) Consider the same initial conditions as in Part A, but now an ***inelastic*** collision occurs between $mass_2$ and $mass_3$ as shown so that they stick together after colliding.



3.2-a) Referring to the figure, what initial conditions for the configuration in figure b) are set by the collision? At t=0, just after the collision, what is the initial velocity v_1 of $mass_1$, the initial velocity v_2 of $mass_2$, the initial velocity v_3 of $mass_3$, and the initial velocity v_4 of $mass_4$.

3.2) (CONTINUED)

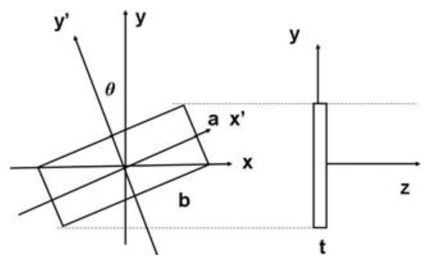
3.2-b) What differential equations govern the motion of the masses? How many DE are there? Write them.

3.2-c) Solve these equations for the position of the masses $x_1(t), x_2(t), x_3(t), x_4(t)$.

3.2-d) Describe the motion qualitatively. How does this motion differ from that in part 3.1? What is the position of the center of the 4 mass system with time? How do the masses respond with time?

Question 4 15 points

Consider a thin plate of width b, height a and thickness t. The x', y' axis are oriented through the centers of the plate sides as shown; the x, y axis are aligned so that the x axis passes as a diagonal through the two farthest points of the plate as shown.
a) What is the moment of inertia about the x' and y' axis. Are the x' and y' axes principal axes? How do you know?



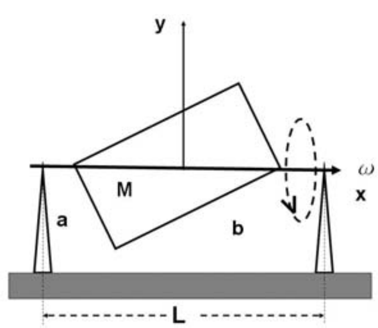
b) For general values of a and b, are the x and y axes principal axes? How do you know?

c) Are there special values for a and b for which the x and y axis are principal axes? How do you know? For this case, what is the complete set of principal axes in the x y plane?

The inertia tensor for the tipped plate in the x, y, z axes system is

$$[I] = M/(12) \begin{pmatrix} 2a^2b^2/(a^2+b^2) & ab(a^2-b^2)/(a^2+b^2) & 0 \\ ab(a^2-b^2)/(a^2+b^2) & (a^4+b^4)/(a^2+b^2) & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix}. \quad (2)$$

The rectangular plate is fixed to a massless rod through its center of mass along the x axis as shown. The rod is supported at two points separated by a distance L as shown, equally spaced from the center of mass. The plate rotates with the rod with an angular velocity $\omega\mathbf{i}$.



d) What is the angular momentum of the plate? Make a sketch of the angular momentum vector relative to inertial space. What is the time rate of change of angular momentum?

e) What forces must be exerted by the supports? Give a formula for the forces in the two supports as a function of time.

f) If the rod were not massless but had a mass M_r, a radius R_0 and a length L_2, how would your answer change?

Question 5 15 points

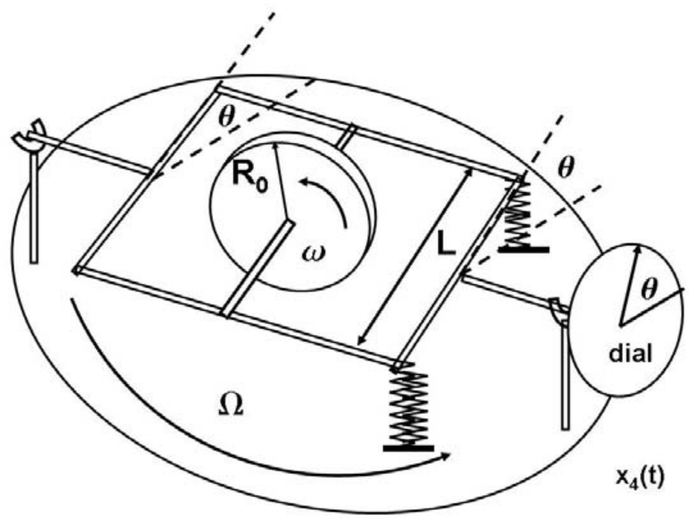
A spinning gyroscope is supported by a gimbal which produces no torque about the gyro axis. The gyro is a solid circular disk of radius R_0, mass M. The gyro spin angular velocity is ω. The gimbal is supported in a frictionless holder and which in turn is supported on a horizontal turnable at rest. The gimbal axis passes through the center of mass of the gyroscope/gimbal combination. The gimbal is restrained by two springs of spring constant k attached to its ends a distance L apart equally spaced from the gimbal axis. These springs have a equilibrium position $\theta = 0$, where θ is the angle measured from the horizontal through which the gyro is free to tip; assume small $\theta << 1$. When the turntable is **at rest**, the spin axis of the gyro is **horizontal** and the plane of the gimbal is **horizontal**.

a) The moment of inertia of a disc spinning about its axis is $I_0 = 1/2MR_0^2$. Sketch the initial angular momentum vector of the gyro.

b) Then the turntable begins to spin at angular velocity Ω, where $\Omega << \omega$, about the vertical axis. After some settling time, what angle θ does the gyro axis assume with the horizontal plane?

c) Sketch the angular momentum vector and its behavior with time.

d) What moment/torque is required to produce this motion? How is this moment generated?



e) What does the gyro measure? How can it be used as a measuring instrument?