



NAME :

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 11

Out date: Nov. 14, 2007

Due date: Nov. 21, 2007

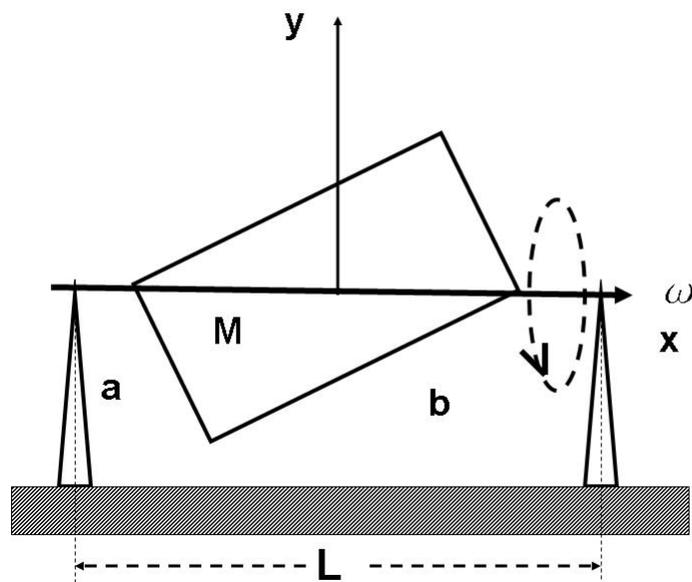
	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Study Time	

Turn in each problem on separate sheets so that grading can be done in parallel

Problem 1 (10 points)

Question 1 10 points

A rectangular plate of mass M , length b and height a is fixed to a massless rod through its center of mass and the corners of the plate as shown. The rod is supported at two points separated by a distance L as shown. The plate rotates with the rod with an angular velocity ω .

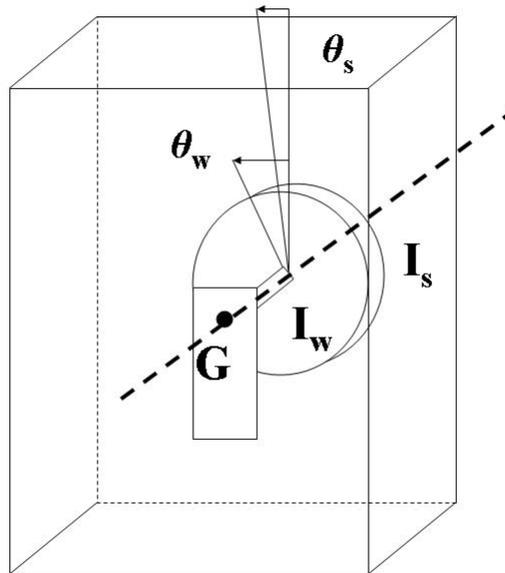


- Is the axis of rotation a principal axis?
- What is the angular momentum of the plate? Make a sketch of the angular momentum vector relative to the position of the plate.
- What forces must be exerted by the supports? Give a formula for the forces in the two supports as a function of time.
- If the rod were not massless but had a mass M_r , a radius R_0 and a length L_2 , how would your answer change?

Problem 2 (5 Points)

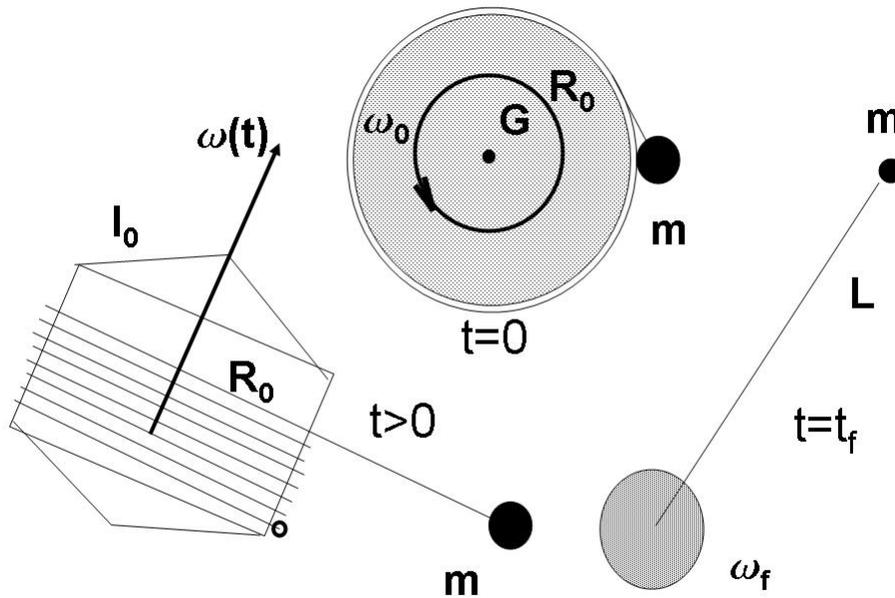
Control of angular orientation and angular momentum are important in satellite operation.

a) Consider first, satellite orientation. Satellite orientation is controlled by internal "momentum wheels". These are discs mounted and connected to the body of the satellite by an electric motor (driven by solar cells.) For simplicity consider the momentum wheel to be mounted at the center of mass of the satellite with its axis along a principal axis of the satellite. If the moment of inertia of the satellite along this axis is I_S , and the moment of inertia of the momentum wheel is I_W , what is the relation between the angular change of the momentum wheel $\Delta\theta_W$ and the angular change of the satellite $\Delta\theta_S$. If the satellite arrives in orbit after launch and it is upside down, how can it be turned over ($\Delta\theta_S = \pi$)?



b) Now consider control of angular momentum. A satellite arrives in orbit with an initial spin about the vertical axis to stabilize the launch and insertion process. However, its operating mode requires it to have only 1% of its initial angular momentum about the vertical. Fortunately, our engineers have planned for this and have attached a small point mass m , to a string of length L which is wrapped around the circumference of the satellite. At $t = 0$, the latch holding the mass fixed to the satellite is released and the string and mass can move away from the satellite. The mass of the satellite is M , the moment of inertia is I_G .

a) What happens?

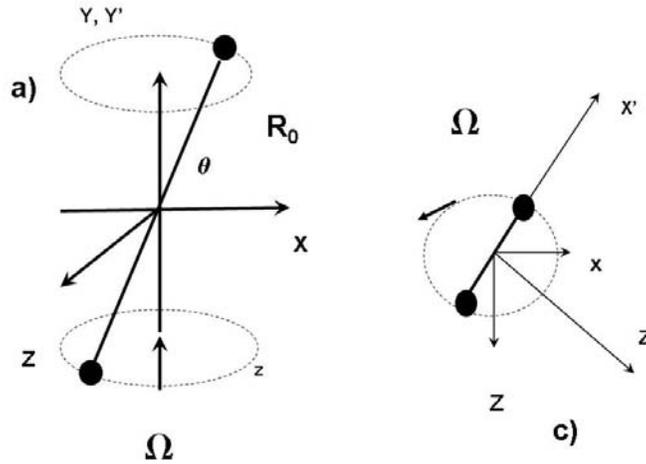


The string reaches its full extent when the *satellite* angular momentum reaches 1% of its initial value. The final configuration is sketched.

b) When the string has reached its full extent as sketched, a second latch releases and the string and mass are set free. What string length is required to reduce the angular momentum of the satellite by 99%?

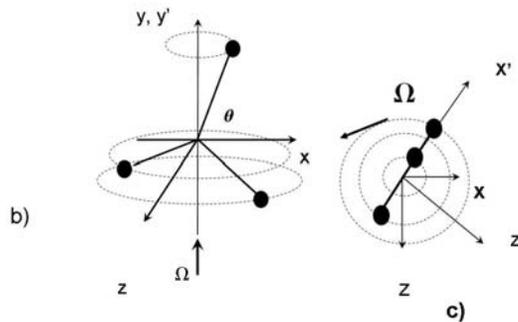
Problem 3(20 points)

Consider the rotation about the y axis of a dumbbell configuration, two mass points m connected by a stiff but massless rod of radius R_0 .



a) Configuration a) has two masses m_a and the rod is oriented in the rotating $x' y'$ plane at a fixed angle θ from the axis of rotation y' . It rotates about the y' axis with angular velocity Ω . Using rotating axis as shown in c), derive an expression for the angular momentum \vec{H} as seen by a fixed inertial observer expressed in the x, y, z system. Sketch the behavior of angular momentum vector with time in the x, y, z coordinate system.

b) What is the rate of change of angular momentum as seen by the inertial observer expressed in the x, y, z system. What torque is required to produce the observed motion in the rotating x', y', z' system? Sketch/plot the components of torque in the rotating coordinate system as a function of the fixed angle θ . Interpret your result physically in terms of the "forces" acting on the masses in the rotating x', y', z' system. Explain why is there a torque in terms of the accelerations of the point masses.



c) Repeat this analysis for the configuration shown in b). This configuration has three masses making equal angles about the circumference and rotates about the y axis with angular velocity Ω . The angle

to the first arm is θ , as in a); the other arms are equally spaced in angle. Take the masses m_b to be $2/3m_a$ (so that the total mass is the same). Discuss the components of angular momentum, rate of change of angular momentum, and torque required to produce the motion, in the rotating x', y', z' system. Comment on, explain and interpret your results including any appropriate references to a).

d) Now consider configurations a) and b) rotating about both the y' and z' axis, by taking $\theta = \omega t$; the angular velocity about y' remains Ω . Derive expressions for the time rate of change of angular momentum for both configurations-and thus the torque required to produce the observed motion as seen in the rotating frame. Comment on the difference between the two configurations with respect to the required torque to produce the prescribed motion. If you were designing the mounts for the motors which had to simultaneously rotate these two configurations about the y' and z' axis, what would you have to worry about? Which design is easier?

e) Use your analysis to comment on the behavior of two- and three- bladed wind turbines. From considerations of aerodynamic efficiency, cost, complexity, it would seem that two blades would be preferred. Both two and three blade would be happy to rotate smoothly about a fixed axis z and deliver power. Right?

However, the wind direction is always changing. So wind turbines must be designed to yaw, that is to rotate about the vertical axis (y') to remain normal to the wind. This problem provides a simple model of the torques required to yaw the wind turbine with angular velocity Ω to the new wind direction while its blades are rotating with angular velocity ω .

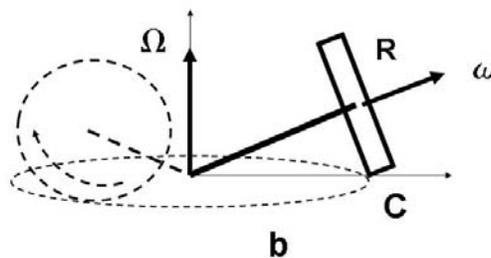
Applying the results of your analysis of the two and three bladed dumbbells, comment on the difference between a two- and three-bladed wind turbine in its yaw behavior. How would the required torques be generated and what effect would they have on the supporting structure? Don't see many two-bladed wind turbines, do you?

How 'bout propellers, of high-performance historically-interesting fighters like the P-51? How 'bout changing flight direction with a two-bladed propeller? What would happen?

Problem 4(5 points)

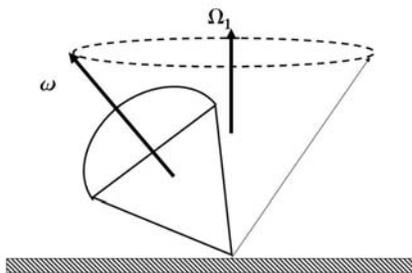
4-A) A solid disk attached to an axis rolls in a circle as shown. the axle is pinned to the ground, and the disk is tipped as shown. The radius of the disk is R ; the distance between the point of contact and the center of the circle is b . The disk rolls without slipping.

- a) What are the principal axes?
- b) Given ω , what is Ω
- c) What is the angular momentum about the point of contact C ?



- d) What is the instantaneous axis of rotation?
- e) Sketch the "body cone"
- f) Sketch the space cone.

4-B) A top spins with angular velocity ω about its axis. The axis of the top traces a circular path with angular velocity Ω .



- a) What are the principal axis?
- b) What is the instantaneous axis of rotation?
- c) Sketch the body cone.
- d) Sketch the space cone.

MIT OpenCourseWare
<http://ocw.mit.edu>

16.07 Dynamics
Fall 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.