16.06 Principles of Automatic Control
Lecture 7

Effects of Zeros on Step Response

We've looked at the response of a second-order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n \mathrm{s} + \omega_n^2}$$

What if we had a zero in the numerator? How would that change the response? Consider:

$$G(s) = \frac{(\alpha s + 1)\omega_n^2}{s^2 + 2\zeta\omega_n \mathrm{s} + \omega_n^2}$$

The step response is then the inverse LT of

$$H(s) = \frac{1}{s}\frac{(\alpha s + 1)\omega_n^2}{s^2 + 2\zeta\omega_n \mathrm{s} + \omega_n^2} = H_0(s) + \alpha s H_0(s)$$

H_0 is

$$H_0(s) = \frac{1}{s}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n \mathrm{s} + \omega_n^2}$$

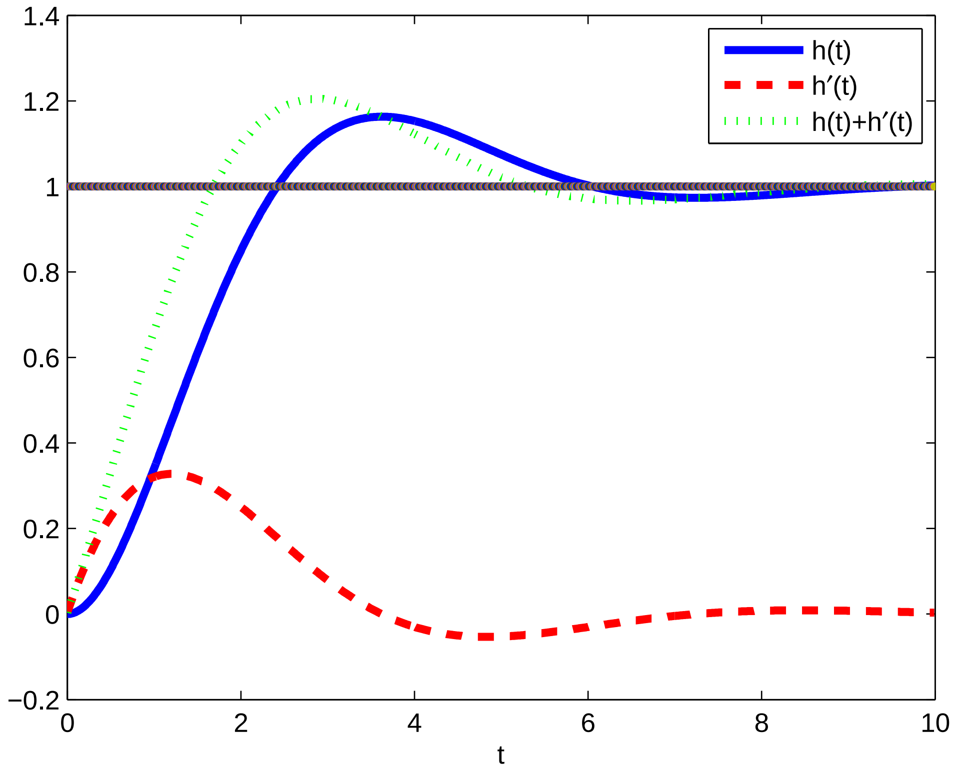
that is, the LT of the step response of the second order system *without* the zero.
The second term is a constant (α) times s times $H_0(s)$. Since multiplying by s is the same as differentiating in the time domain, we have that

$$h(t) = h_0(t) + \alpha \frac{d}{dt} h_0(t)$$

where $h(t)$ is a step response with zero, and $h_0(t)$ is a step response without the zero.

Example:

$$G_0(s) = \frac{1}{s^2 + 2\zeta s + 1}, \zeta = 0.5$$



So a zero (with $\alpha > 0$) tends to speed up time response increase overshoot.
Note that the zero is at root of

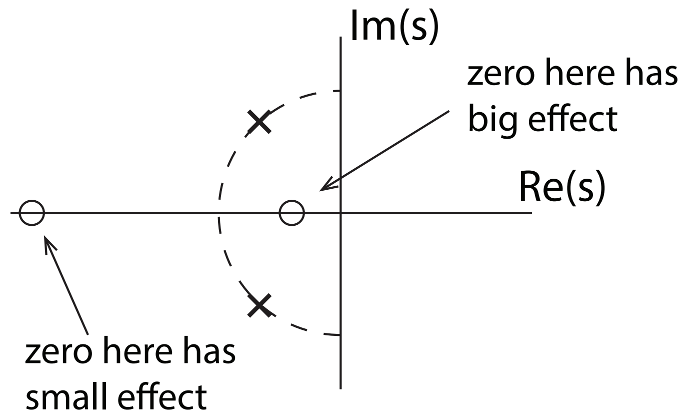
$$\alpha s + 1 = 0 \rightarrow s = -1/\alpha$$

The effect is small if

$$|\alpha \omega_n| \ll 1$$

which is the same as

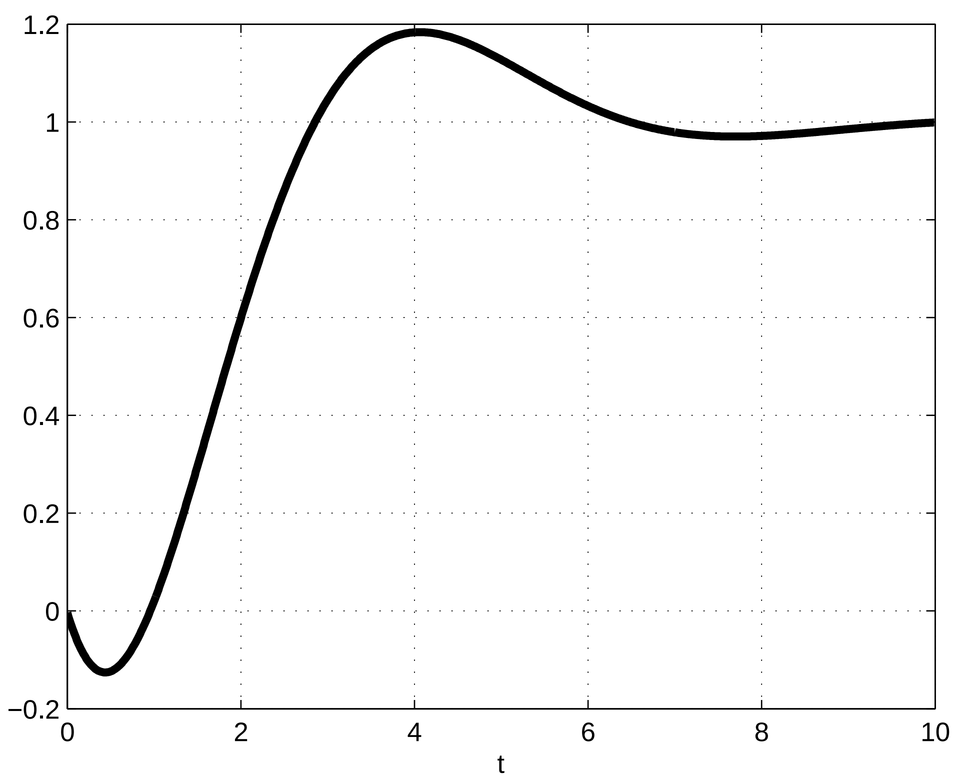
$$|-1/\alpha| \gg |\omega_n|$$



Non-Minimum Phase Zeros

For technical reasons, a zero in the right half plane are called “non-minimum phase zeros”. They have a funny (undesirable) effect on the response of a system.

For a NMP zero, $\alpha < 0$. So, the step response will be the original step response *minus* a constant times the derivative:



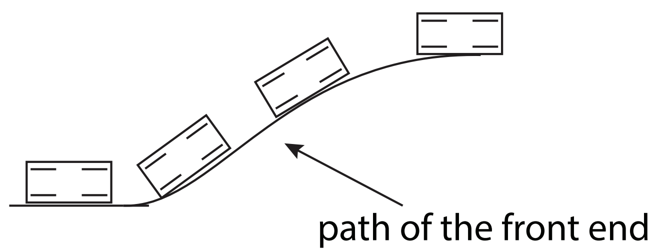
This results in initial "wrong way" behavior that is *very* undesirable.

Examples of NMP systems:

1. Space shuttle on approach

2. Backing car

To get a *backing* car to return to a given line, must make the front end go the wrong way first:



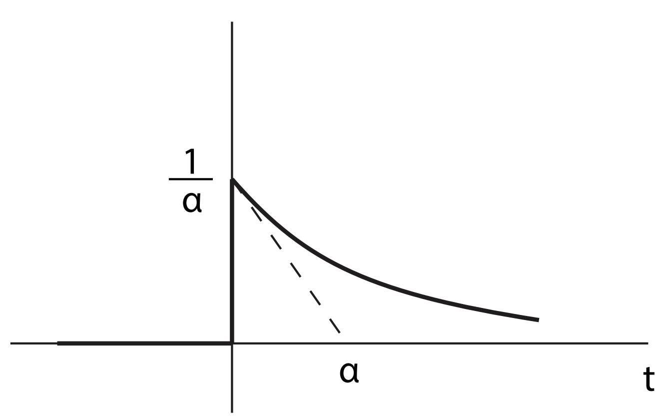
Effect of Additional Poles

What happens if we add a third pole?

$$G(s) = \frac{1}{\alpha s + 1}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The new step response will be the convolution of the step response without the additional pole with:

$$\frac{1}{\alpha}e^{-\frac{t}{\alpha}}\sigma(t)$$



So, net effect is that addition of the pole will smooth the original step response, increasing the rise time, and reducing the overshoot,