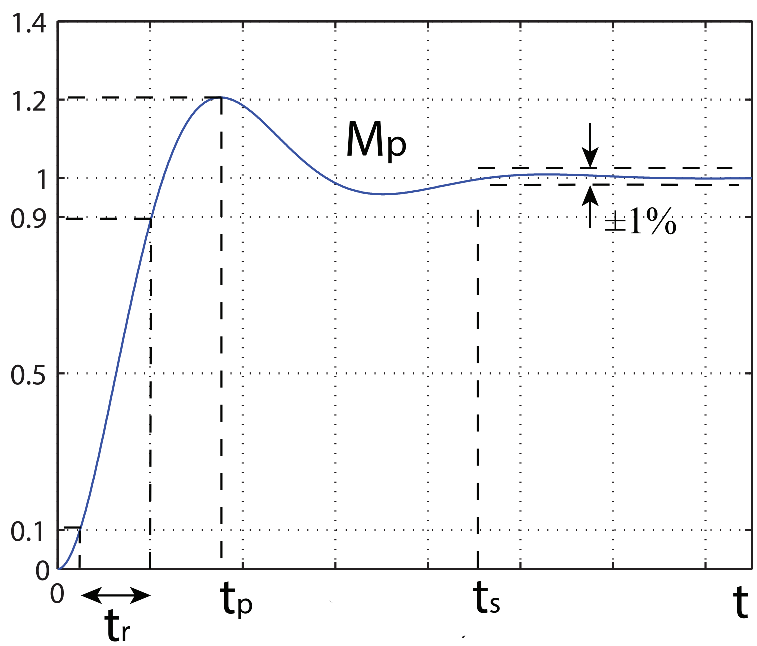
16.06 Principles of Automatic Control
Lecture 6

Time Domain Specifications:

Many control systems are dominated by a second order pair of poles. So look at time response (to step input) of

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Typical response:



$$
\begin{aligned}
M_p = & \quad \text{peak overshoot} \\
t_r = & \quad \text{rise time (10\% to 90\%)} \\
t_s = & \quad \text{settling time (1\%)} \\
t_p = & \quad \text{time of peak}
\end{aligned}
$$

Each of the above parameters may be important in the design of the control system. For example, the designer of a hard disk drive may specify a maximum settling time of the response of the read/write head to a commanded change in position.

Peak overshoot is important, both because it is a measure (to a degree) of stability, and for practical reasons, overshoot should be minimized (think of an elevator!).

Rise time t_r(and to a lesser extent peak time t_p) is a measure of the speed of response of the system. Often, a maximum $t_r$will be specified.

We can connect $\zeta$and $\omega_n$to M_p, t_p, t_r, with two important caveats: first, some of the relationships are approximate. Second, additional poles and zeros will change the results, so all of the results should be viewed as guidelines.

The step response of $H(s)$ is

$$h_s(t) = 1 - e^{-\zeta\omega_n t}\left(\cos(\omega_d t) + \frac{1}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right)$$

Using elementary calculus, we can find t_p and M_p(see text):

$$
\begin{aligned}
t_p =& \frac{\pi}{\omega_d} \\
M_p =& e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \\
=& e^{-\pi \tan\Theta}
\end{aligned}
$$

where $\Theta = \sin^{-1}\zeta$.

Typical values:

ζ	M_p
0.5	0.16
0.7	0.05

The rise time is approximately

$$t_r \approx \frac{1.8}{\omega_n}$$

The *rise time* is a bit faster for systems with less damping, a bit longer for systems with more damping, and sensitive to additional poles and zeros.

The *settling time* can be approximated via:

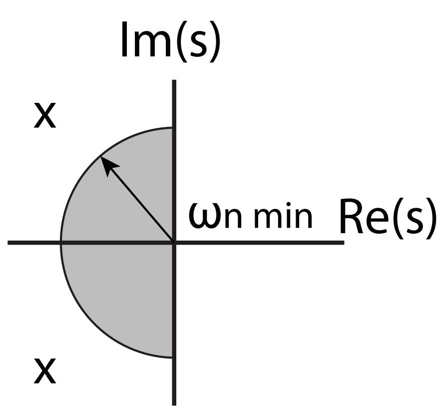
$$\begin{aligned} e^{-\zeta\omega_n t_s} &\approx 0.01 \\ \rightarrow t_s &\approx \frac{4.6}{\zeta\omega_n} \end{aligned}$$

Note that, in reality, settling time varies discontinuously with ζ, since as damping increases, a peak may decrease from just over 1.01 to just under 1.01, so $t_s$is drastically reduced.

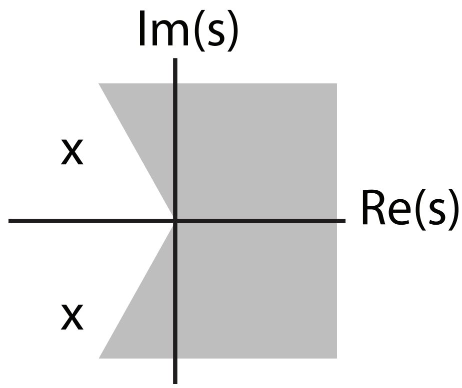
Desired pole locations

Given specifications on t_r, M_p, and t_s, where should poles be?

$$\begin{aligned} t_r &\leqslant a \\ \rightarrow \frac{1.8}{\omega_n} &\leqslant a \\ \rightarrow \omega_n &\geqslant \frac{1.8}{a} = \omega_{n_{min}} \end{aligned}$$

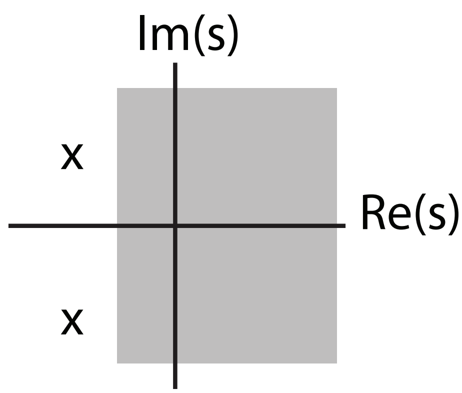


Likewise, to keep $M_p$less than a fixed value, must have $\zeta \geqslant \zeta(M_p)$:

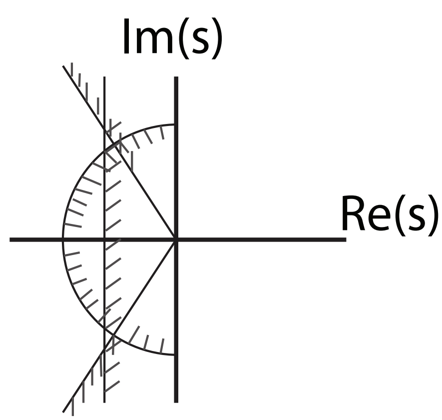


Finally, must have:

$$\zeta\omega_n \geqslant \frac{4.6}{t_s}$$



Putting these constraints together will yield an allowable region for the poles (see better drawing in text):



N.B.: The allowable region is a *guide*. After a system is designed, the performance will have to be evaluated.

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