16.06 Principles of Automatic Control
Lecture 5

Dynamic Response:

Usually, we find the response of a system using Laplace techniques. Often, do within Matlab.

Example: DC Motor.

Suppose:

$J = 0.01$ kg·m^2; $b = 0.001$ N-m-sec

$K_t = K_e = 1$ n-M/A $= 1$ V/(rad/sec)

$R_a = 10\Omega,\ L = 1$ H

Then

$$\begin{aligned}
\frac{\Theta}{V_a}(s) &= \frac{100}{s^3 + 10.1s^2 + 101s} \\
\frac{\Omega}{V_a}(s) &= \frac{s\Theta}{V_a}(s) = \frac{100s}{s^3 + 10.1s^2 + 101s} \\
&= \frac{100}{s^2 + 10.1s + 101} \\
G(s) &= \frac{100}{(s + 5.05 + j8.6889)(s + 5.05 + -j8.6889)}
\end{aligned}$$

What is the step response of the motor? That is, what is the velocity of the motor as a function of time, if $v_a(t) = \sigma(t)$?

By hand, would do:

$$
\begin{aligned}
g_s(t) &= L^{-1}\left[\frac{1}{s}G(s)\right] \\
\frac{1}{s}G(s) &= \frac{100}{s(s+a+jb)(s+a-jb)} \\
&= \frac{r_1}{s} + \frac{r_2}{s+a+jb} + \frac{r_3}{s+a-jb}
\end{aligned}
$$

Would find r_1, r_2, r_3 by partial fraction expansion.
Then find $L^{-1}$of each term, add together, and simplify. A *lot* of work.

Instead, use MATLAB:

```
num=[0 0 100];
den=[1 10.1 101];
sysg=tf(num,den);
t=0:0.01:5;
y=step(sysg,t);
plot(t,y);
```

The above code produces the following figure:

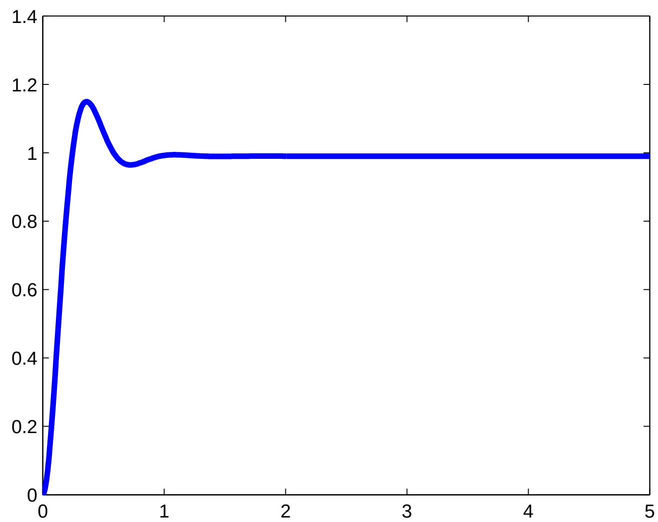


Figure 1: Velocity of the motor.

The above system was an open-loop system. Would do the same for a closed-loop system, after finding the transfer function.

Example:

The transfer function from aileron input (δ_a) to roll angle (ϕ) is given by

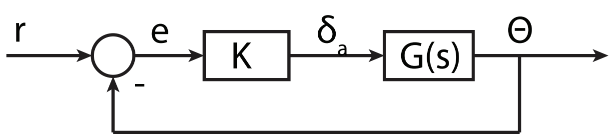
$$\frac{\Phi}{\delta_a}(s) = \frac{k}{s(\tau s + 1)}$$

$$\begin{aligned} \text{where } k &= \text{steady roll-rate per unit of aileron deflection} \\ \tau &= \quad \text{roll subsidence time constant} \\ &= \frac{I}{-M_{\dot{\phi}}} \end{aligned}$$

Suppose δ_a is measured in % of full deflection, so $\delta_a = 1$ is full right aileron, $\delta_a = -1$ if full left one. Then a typical set of parameters might be

$$\begin{aligned} k &= 100 \text{ deg/sec} \\ \tau &= 0.5 \text{ sec} \\ G(s) &= \frac{100}{s(0.5s + 1)} \end{aligned}$$

Suppose we implement the following control law:



What is the transfer function of a closed-loop system?

$$\begin{aligned} H(s) &= \frac{KG(s)}{1 + KG(s)} = \frac{\frac{Kk}{s(\tau s+1)}}{1 + \frac{Kk}{s(\tau s+1)}} \\ &= \frac{Kk}{\tau s^2 + s + Kk} \end{aligned}$$

Suppose we take $K = 0.1/\text{deg}$.

Then:

$$H(s) = \frac{10}{0.5s^2 + s + 10}$$
$$H(s) = \frac{20}{s^2 + 2s + 20}$$

Find step response via MATLAB:

```
num=[0 0 20];
den=[1 2 20];
sysg=tf(num,den);
t=0:0.01:5;
y=step(sysg,t);
plot(t,y);
xlabel('Time, t (sec)');
ylabel('Roll angle, \phi (deg)');
```

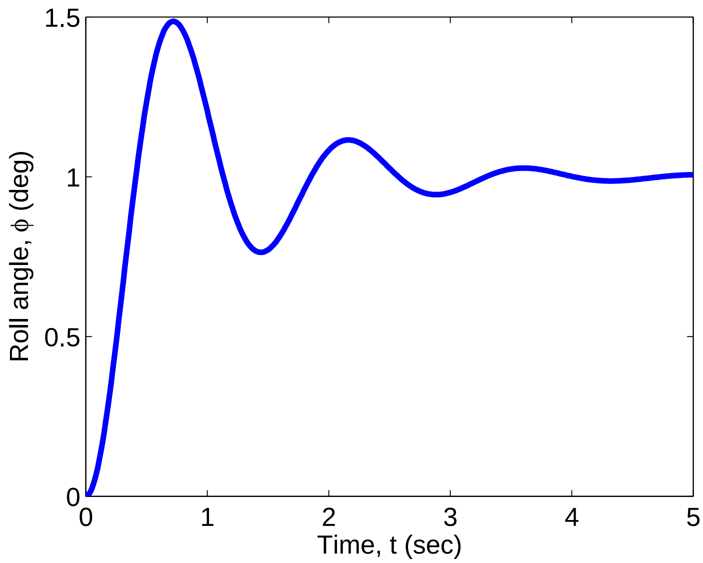


Figure 2: Roll angle vs time.

The result (shown in Figure 2 is NOT very good. Oscillatory!

More on this later.

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