16.06 Principles of Automatic Control
Lecture 34

Zeros:	$z = -0.9867..$	$\rightarrow$	$W = -30,000$
	$z = \infty$	$\rightarrow$	$W = +200$
Poles:	$z = 0.9802$	$\rightarrow$	$W = -2$

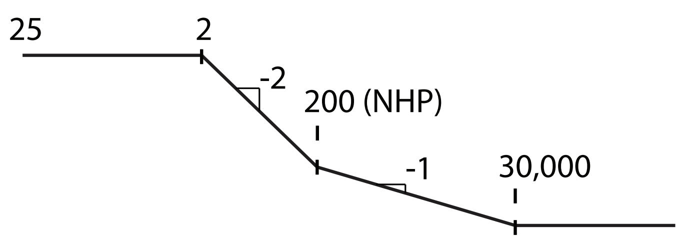
Bode Gain: $G_d(1) = 25$.

Also, need to map crossover frequency:

$$\nu_c = \frac{\tan(\omega_c T/2)}{T/2} = 51.068$$

There's not much warping at $\omega T = 0.5$ (2%). At $\omega T = 1.0$, there is 9% warping; at $\omega T = 1.5$, 24%. So for most problems, may not need to prewarp.

Bode plot of G:



To meet specs, need lead around $\nu_c = 51$, lag below $\nu = 5.1$.

$$\angle G(j51) = -189.7^\circ$$

So need phase from lead compensator

$$\begin{aligned}\phi_{\text{lead}} &= -180^\circ - \angle G(j51) + 6^\circ + \text{PM} \\ &= 65.7^\circ\end{aligned}$$

where 6° anticipates lag compensation.

$$\begin{aligned}\Rightarrow \sqrt{\frac{b}{a}} =& 4.65\\ \Rightarrow b =& 51 \cdot 4.65 = 237\\ a =& 51/4.65 = 11.0\\ \Rightarrow K_{\text{lead}} =& 5.44\ \frac{1 + W/11}{1 + W/237}\end{aligned}$$

For this compensator, $K_p = 136$. So need lag ratio of 36.76.

$$\Rightarrow K_{\text{lag}} = \frac{W + 5}{W + 0.136}$$

Therefore, the compensator is

$$K(W) = 5.44\ \frac{W + 5}{W + 0.136}\ \frac{1 + W/11}{1 + W/237}$$

The discrete time compensator is the Tustin transform, yielding

$$K_d(z) = 57.97\ \frac{(z - 0.9512)(z - 0.8957)}{(z - 0.9986)(z + 0.0847)}$$

Remarks:

1. Compensator is almost identical to what would be found if we used emulation, *if* we include effective $T/2$ delay of ZOH.

2. W-Transform approach guarantees stability of discrete-time system. Not really an issue for $\omega_c T = 0.5$, but might be for faster crossover.

3. Note RHP zero at $wW = +\frac{2}{T} = 200$. This zero limits achievable bandwidth of controller, just like time delay.

Example

Same plant as above. Make ω_c (ν_c) as high as practical and still have 50°phase margin.

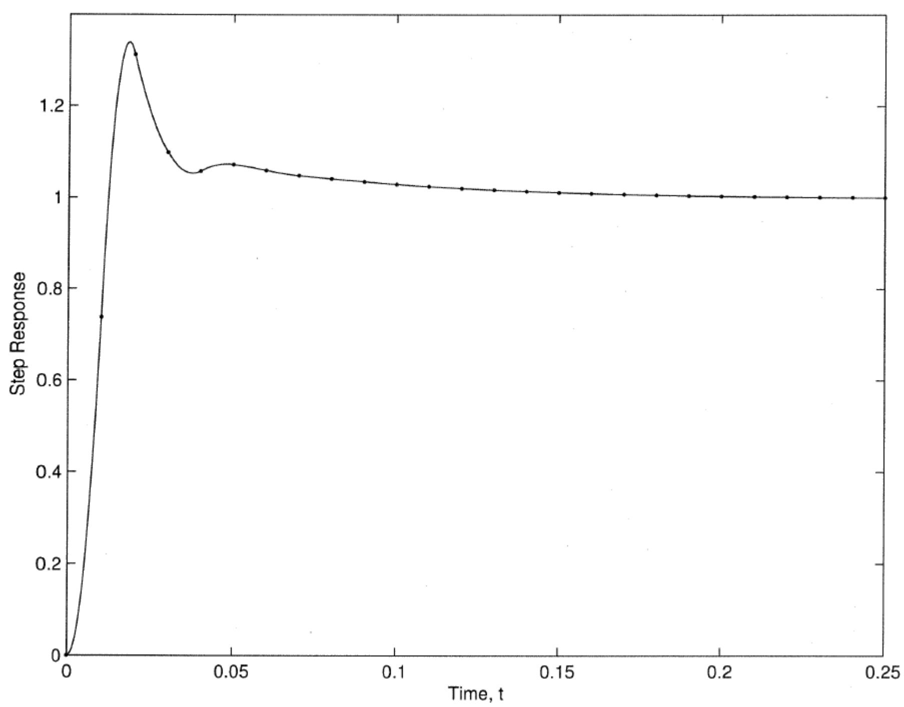
Solution: Let's pick crossover frequency ν_c to be factor of only 2 below NMP zero.

$$\nu_c = 100$$

Use lead compensator to get desired PM and crossover:

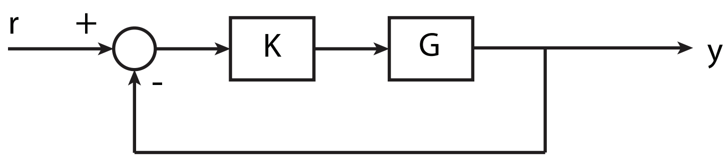
$$
\begin{aligned}
\angle G_w(j\omega_0) &= -204.1^\circ \\
\phi_{\text{lead}} &= -180 - \angle G + \text{PM} = 74.1^\circ \\
\Rightarrow \sqrt{\frac{b}{a}} &= 7.15 \\
b &= 100 \cdot 7.15 = 715 \\
a &= 100/7.15 = 14 \\
\Rightarrow K_N(W) &= 12.53\ \frac{1 + W/14}{1 + W/715} \\
\Rightarrow K_d(z) &= 149.7\ \frac{z - 0.8692}{z + 0.5628}
\end{aligned}
$$

See step response below:



Direct Design

Suppose we have the usual unity feedback control structure:



(The system might be continuous or discrete). Suppose we want the closed loop transfer function

$$H = \frac{KG}{1 + KG}$$

to have a specific form, e.g., have a particular rise time, settling time, etc. Why not just *solve* for desired K in terms of G, H?

$$\begin{aligned} H(1 + KG) &= KG \\ H &= -KGH + KG \\ K &= \frac{1}{G} \frac{H}{1 - H} \end{aligned}$$

So

$$\boxed{K = \frac{1}{G} \frac{H}{1 - H}}$$

Note that K essentially cancels G with the factor $1/G$, so makes the loop gain

$$K = \frac{H}{1 - H}$$

exactly what is needed to have the desired closed loop transfer function.

But we can't choose any H desired!

Constraints on H:

Stability In order that K not cancel on unstable pole or NMP zero, we must have that

1. H must have as zeros all the zeros of G outside the unit circle.
2. $1 - H$ must have as zeros all the unstable poles of G.

Causality In order that K be causal, we must have:

3. The relative degree of H is at least as large as the relative degree of G.

Example

$$
\begin{aligned}
G(s) &= \frac{25}{(1+s/2)^2} \\
G_d(z) &= 0.004934 \ \frac{z+0.9867}{(z-0.9802)^2} \\
T &= 0.01
\end{aligned}
$$

do a direct design such that

1. The system is Type 1: $\Rightarrow \quad H(1) = 1$.
2. The system is deadbeat (all poles of H at $z = 0$).

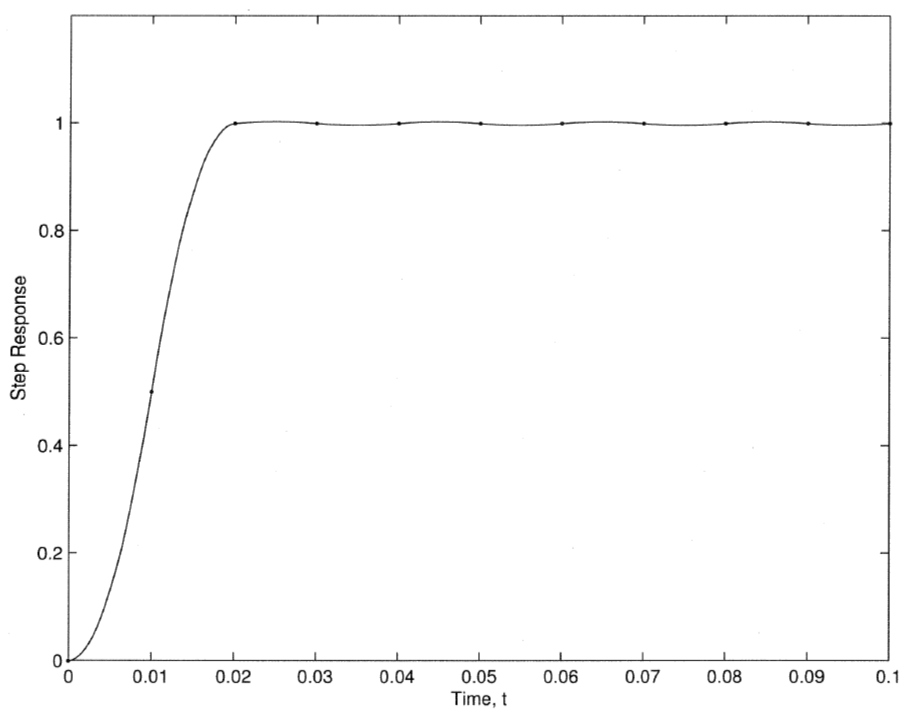
Therefore, might select

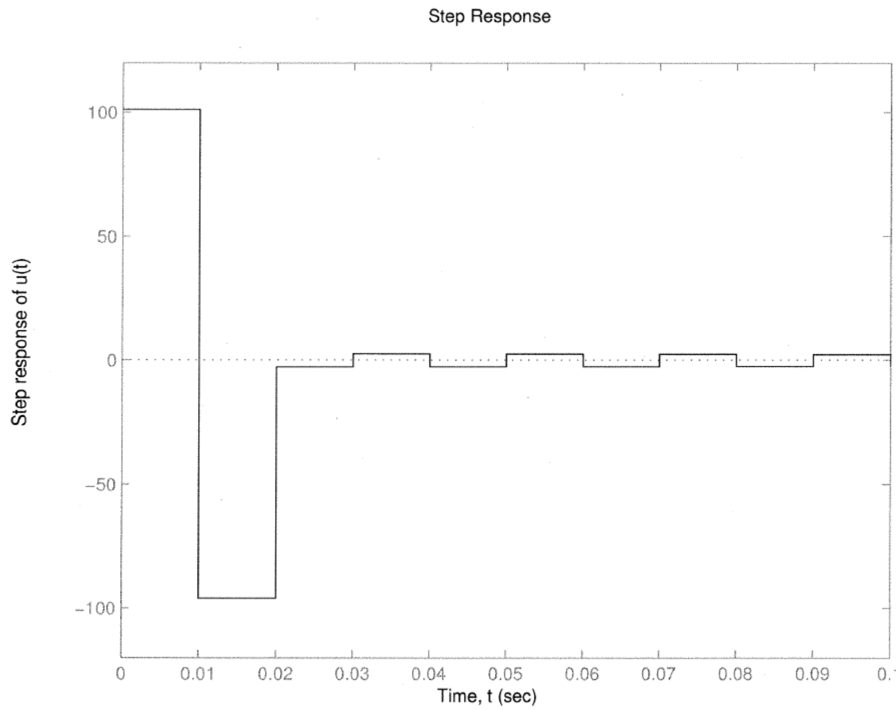
$$H(z) = \frac{1}{2} \ \frac{z+1}{z^2}$$

Then

$$K_d(z) = \frac{1}{G_d(z)} \cdot \frac{1}{2} \cdot \frac{z+1}{z^2 - 0.5z - 0.5}$$

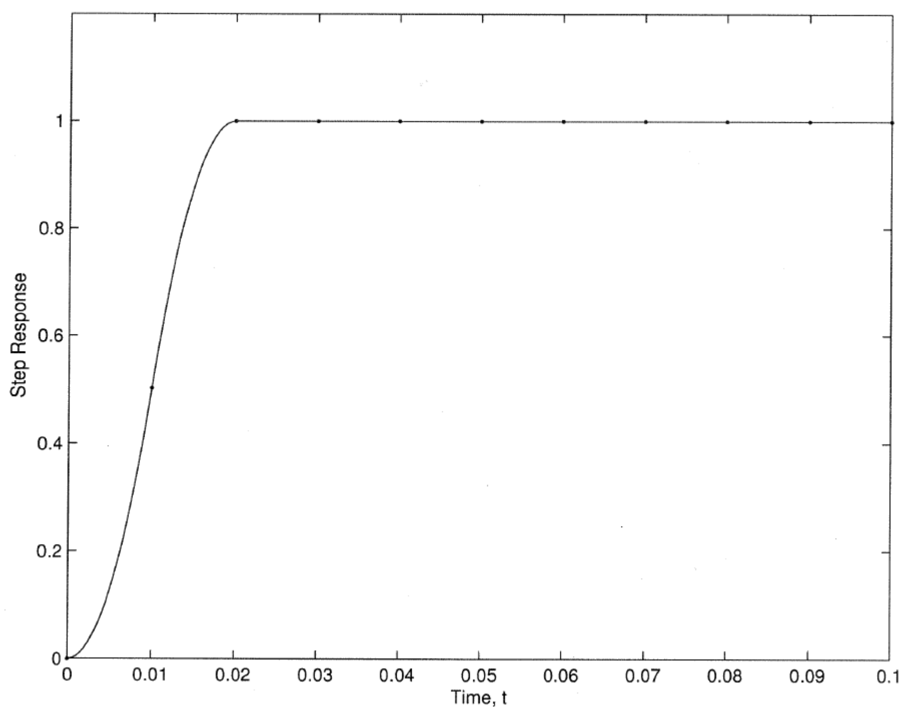
See responses on next two pages. Note the "ringing" in $u[k]$. To eliminate, put zero of H at -0.9867.

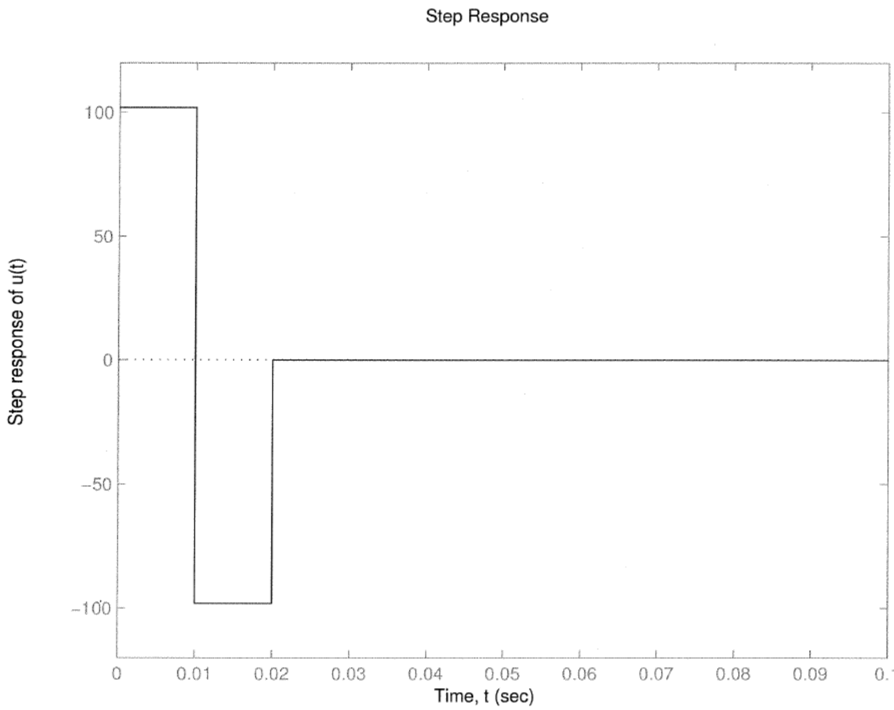




To eliminate ringing, choose

$$H(z) = \frac{1}{1.9867} \frac{z + 0.9867}{z^2}$$





Step Response
Step response of u(t)
100
50
0
−50
−100
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Time, t (sec)