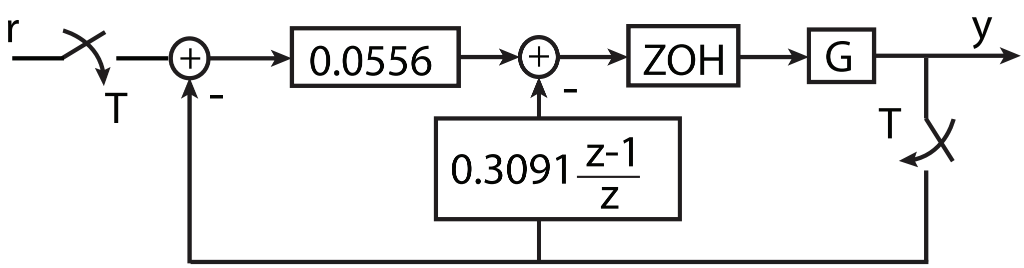
# 16.06 Principles of Automatic Control
# Lecture 33




Note that

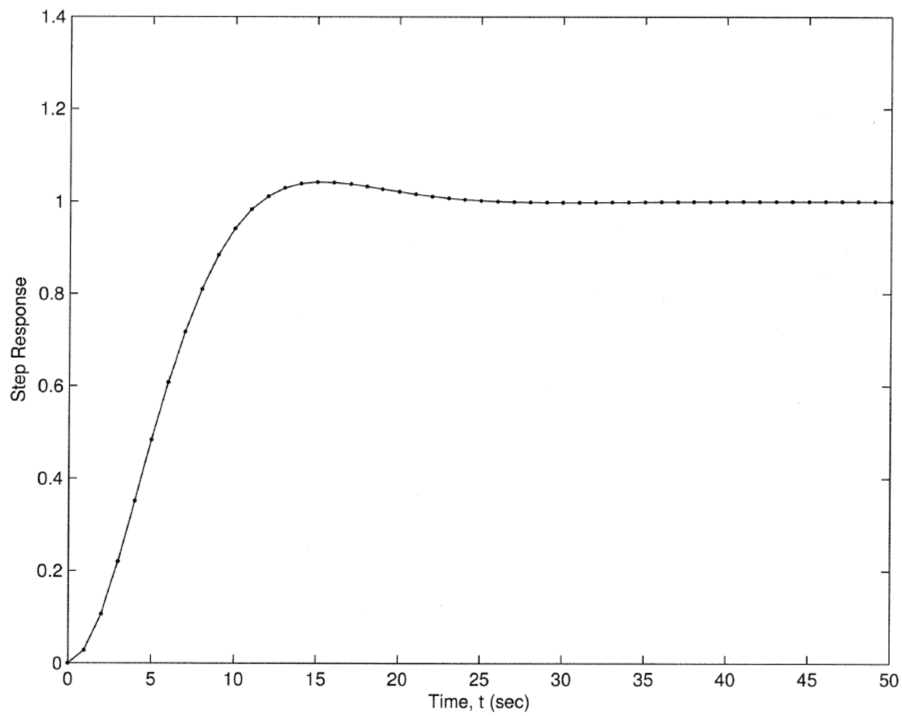
$$K_d(1) = 0.0556 = 1$$

$$K_d(z) - K_d(1) = 0.3091\frac{z-1}{z}$$

The step response is shown below. Note the much improved response. The peak overshoot is

$$M_p = 0.042$$

very close to the ideal $M_p = 0.043$ for $\zeta = 0.7071$.




# Discrete Design vs. Emulation

The text argues that discrete design should be used if

$$\omega_s = \frac{2\pi}{T} < 10\omega_n$$

or

$$\omega_n T > \frac{\pi}{5} \approx 0.63$$

I *disagree.* If the effective time delay is taken into account, emulation works well out to

$$\omega_c T \approx 1$$

and maybe higher. But that is close to the upper limit on how high it is possible to cross over due to $T/2$ time delay.

$$\omega_c \frac{T}{2} \leqslant \begin{cases} 1, & \text{less cons. upper limit} \\ 0.6, & \text{more cons. upper limit.} \end{cases}$$

So emulation should work in all but the most severe cases.

## The $W-$Transform

The $W-$ Transform is used to allow the use of classical continuous time design techniques (including Bode plots) on discrete-time systems.

Recall that the Tustin transform is

$$s \to \frac{2}{T}\frac{z-1}{z+1}$$

$$z \to \frac{1+sT/2}{1-sT/2}$$

So there is no confusion, we use the variable $W$ instead of $s$, so the $W-$transform is

$$z \to \frac{1+WT/2}{1-WT/2}$$

Can show that this mapping between $z$ and $W$:

- Is one to one (unique $W$ for each $z$, vice-verse).
- Maps the unit disk to the left half plane.

Note that the W-transform "warps" frequencies:

$$\begin{aligned} W =& \frac{2}{T}\ \frac{z-1}{z+1} \\ =& \frac{2}{T}\ \frac{e^{j\omega T}-1}{e^{j\omega T}+1} \text{ (on unit circle)} \\ =& \frac{2}{T}\ \frac{e^{+j\omega T/2}-e^{-j\omega T/2}}{e^{+j\omega T/2}+e^{-j\omega T/2}} \text{ (multiply top and bottom by } e^{-j\omega T/2}\text{)} \\ =& \frac{2j}{T}\ \tan\left(\frac{\omega T}{2}\right) \text{ (Trig. identities)} \\ =& j\nu \text{ (}\nu\text{=frequency in W domain)} \end{aligned}$$

Therefore,

$$\nu = \frac{\tan(\omega T/2)}{T/2}$$

where $\omega$ =physical frequency,

$\nu$ =apparent frequency in W.

Example

$$
\begin{aligned}
G_d(z) =& 0.004934 \frac{z + 0.9867}{(z - 0.9802)^2} \\
T =& 0.01
\end{aligned}
$$

Design a controller $K_d(z)$ so that

- $PM = 50^\circ$
- $\omega_c = 50$ r/s
- $K_p = 5000$

First, find $G(W)$ using MATLAB:

```
gw=d2c(gd,'tustin')
```

Result is

$$
G(W) = 25 \frac{(1 + \frac{W}{30{,}000})(1 - \frac{W}{200})}{(1 + W/2)^2}
$$

Zero at $W = -30,000$ can be ignored, but note presence of RHP zero at $W = +200$.

Alternatively, map poles and zeros by

$$
W = \frac{2}{T} \frac{z - 1}{z + 1}
$$