16.06 Principles of Automatic Control
Lecture 27

Nonminimum Phase Systems

Our design rules so far are based on the bode gain-phase theorem, which applies to stable, minimum phase systems. The RHP zeros or time delays of NMP systems place fundamental limitations on the achievable performance of any closed-loop systems.

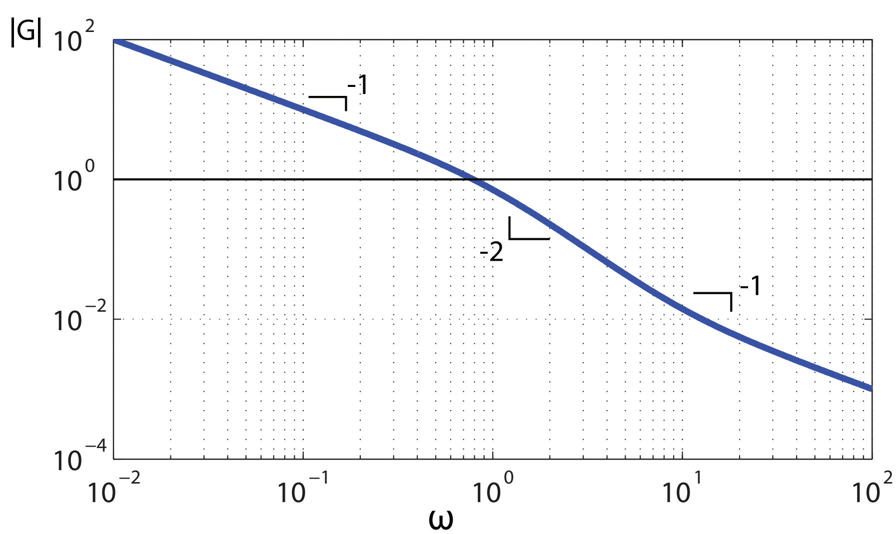
Example:

Consider the plant

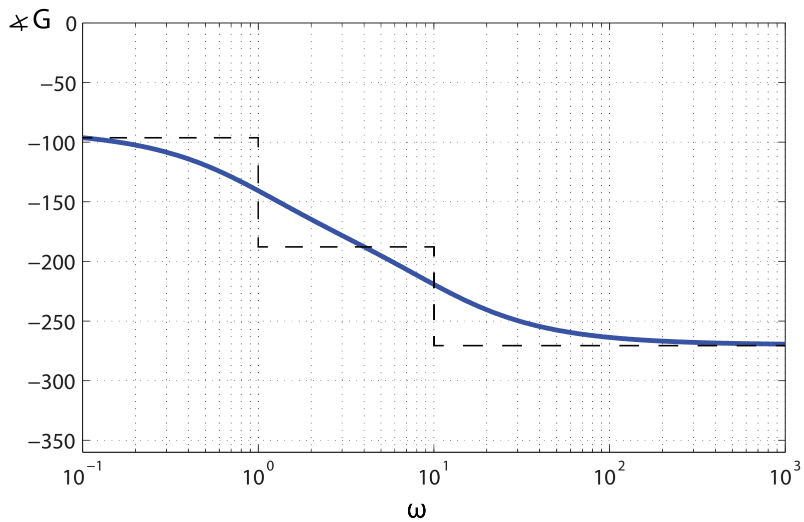
$$G(s) = \frac{1 - s/10}{s(1 + s/1)}$$

Our goal is to design a closed-loop controller with bandwidth as large as possible. How well can we do?

Bode plot:



The slope at high frequency is -1, so it seems that we should be able to cross-over anywhere. However, in this case we need to look at the phase plot, sicne gain-phase theorem does not apply:



Note that additional phase due to zero at $s = +10$ is negative.

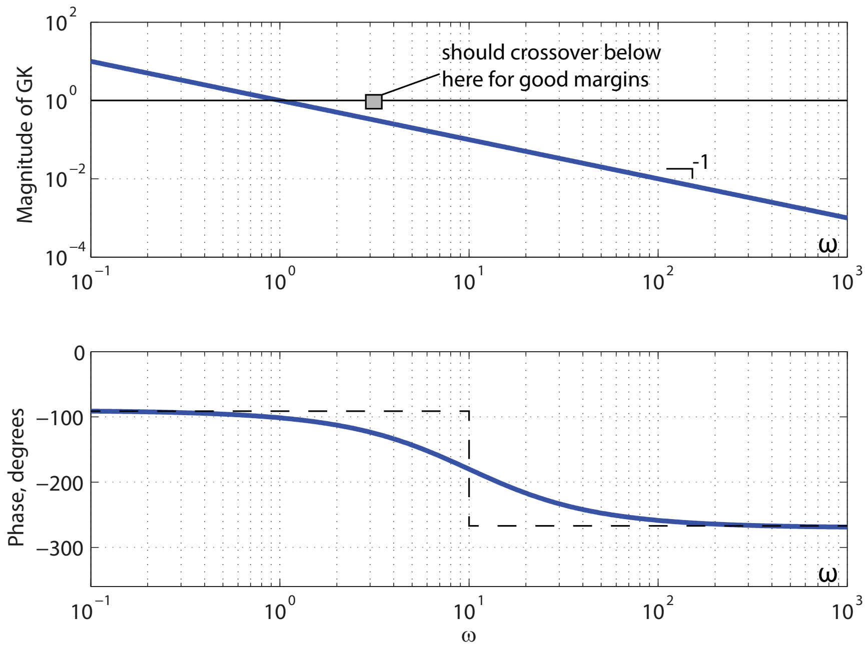
So if we use pure gain, the crossover frequency must be below about $\omega_c = 3$.

Let's add compensation to make slope -1 everywhere:

$$K(s) = k\frac{1 + s/1}{1 + s/10} \begin{array}{l} \leftarrow \text{cancels plant pole} \\ \leftarrow \text{stable pole} \end{array}$$

$$\Rightarrow K(s)G(s) = \frac{1}{s}\frac{1 - s/10}{1 + s/10}$$

Bode Plot:



So NMP zero causes significant phase lag (relative to the phase expected from slope) at frequencies up to one decade below crossover.

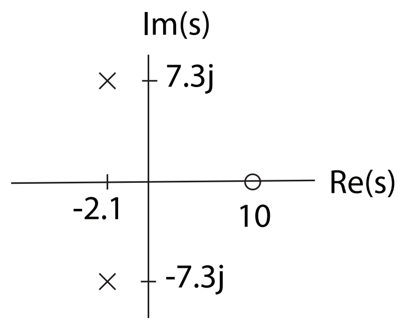
Suppose we could accept PM as low as PM = 30°. What would control system look like?

Solve for k:

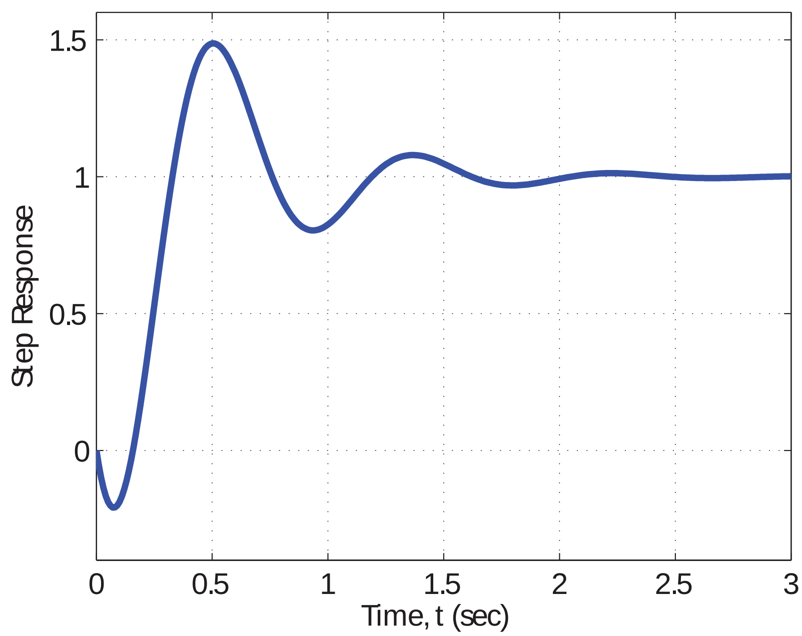
$$
\begin{aligned}
\angle GK =& -90^\circ - 2\tan^{-1}\omega/10 \\
=& -150^\circ \\
\Rightarrow \omega_c =& 5.77 \,\mathrm{r/s} \\
|GK| =& k/\omega \\
k = \omega_c =& 5.77
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
K(s) =& 5.77\,\frac{1+s}{1+s/10} \\
T(s) =& \frac{K(s)G(s)}{1+K(s)G(s)} \\
=& \frac{1-s/10}{\frac{s^2}{57.7} + \frac{2(0.278)s}{\sqrt{57.7}} + 1}
\end{aligned}
$$



See step response plotted below:



Note that $M_p \approx 49\%$.

In addition, there is a 20% *undershoot* (wrong way behavior).

The bottom line is that a non-minimum phase zero places fundamental limitations on the bandwidth of the closed-loop system. As a practical matter, if the NMP zero is at $s = a$, we must have

$$\omega_c \leqslant a/2$$

More realistically, to achieve reasonable phase margins and step response, we need

$$\omega_c \leqslant a/3$$

Even at $\omega = a/10$, the NMP zero adds 12° of anomalous phase lag.

Time Delay

The effect of a pure time delay is similar to that of a NMP zero. Indeed, a time delay *is* non-minimum phase. The transfer function os a T-second delay is

$$e^{-sT} = e^{-j\omega T}$$

So the additional phase lag is ωT. As a practical matter, must cross over at

$$\omega_c \leqslant 1/T$$

but more reasonably should have

$$\omega_c \leqslant 0.6/T$$

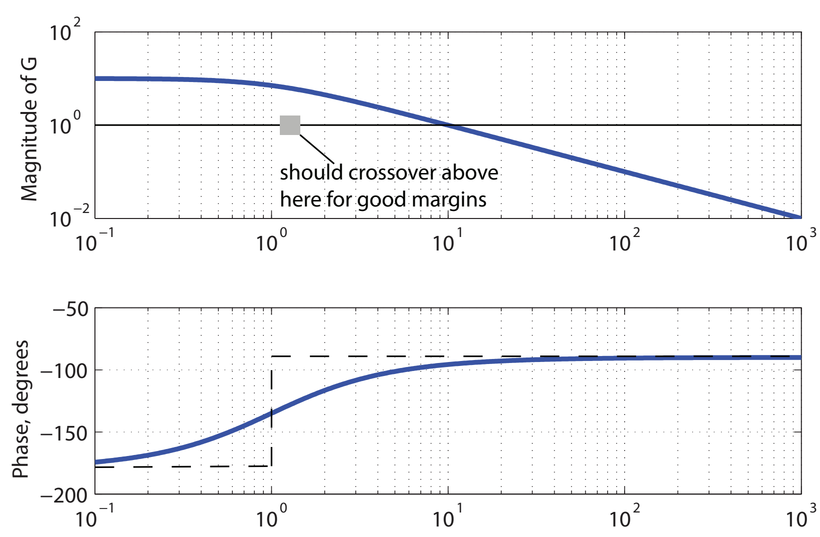
Unstable systems

For an unstable system, the Bode gain-phase theorem does not apply either. In this case, however, the disagreement between slope and phase occurs at low frequency (when viewed properly).

Example:

$$G(s) = \frac{10}{s-1}$$

Bode:



Using arguments similar to those made for NMP zeros, can see that we need to crossover at least at

$$\omega_c \geqslant 2p$$

where p is the location of the unstable pole.

Note that this is a fuzzy requirement - inthe example, can stabilize the system wtih any $\omega_c > 0$, but margins will be poor unless $\omega_c \geqslant 2$ r/s.