16.06 Principles of Automatic Control
Lecture 26

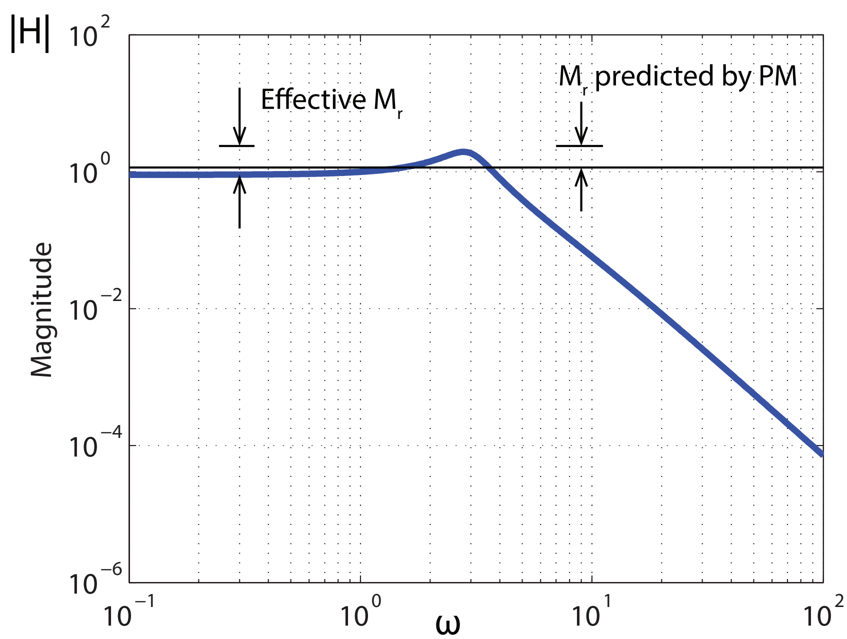
From last time, we had plant and compensator

$$G(s) = \frac{1}{(1 + s/0.5)(1 + s)(1 + s/2)}$$

$$K(s) = 9\,\frac{1 + s}{1 + s/8}$$

The closed-loop step response has 45% overshoot, when 37% expected. Why?

Look at Bode plot of $H = \frac{KG}{(1+KG)}$:



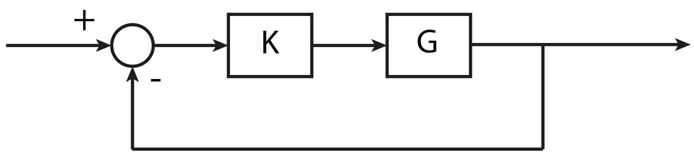
Because of low K_p, D.C. gain of H is 0.9, which increases effective M_r by factor of $1/0.9$.

Lag Compensator

Consider the plant

$$G(s) = \frac{1}{s(s+10)}$$

In a unity feedback control system



Suppose we use a proportional controller

$$K(s) = 141$$

For this controller,

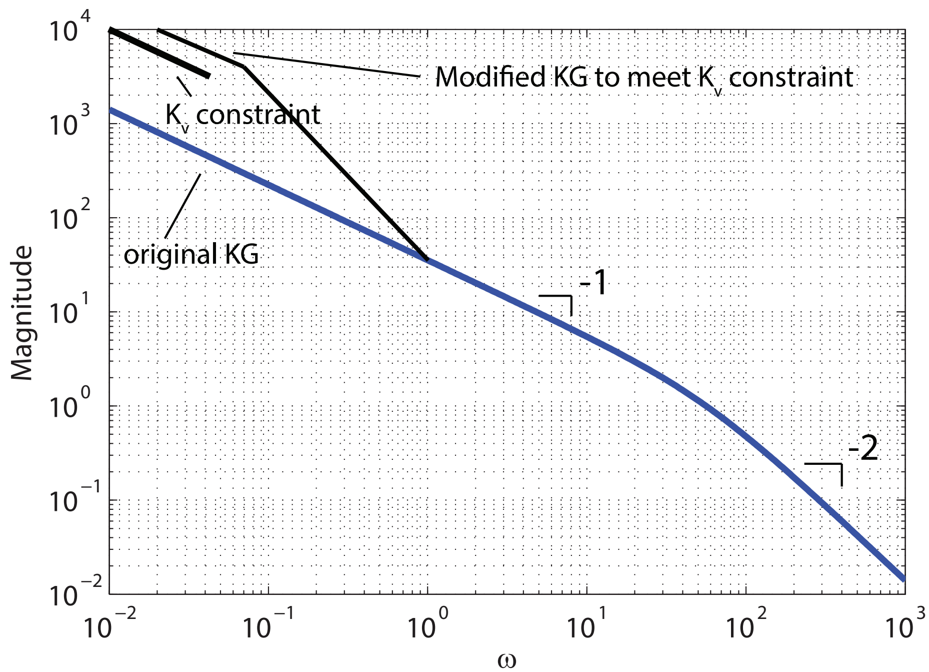
$$\begin{aligned} \omega_c =& 10\,\mathrm{r/s} \\ \mathrm{PM} =& 45^\circ \end{aligned}$$

and the overshoot in response to a unit step is

$$M_p = 23\%$$

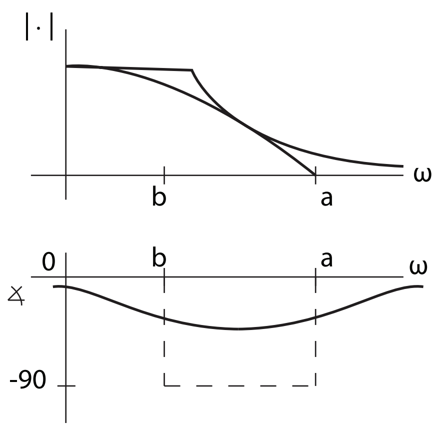
Suppose that we find the response of the closed-loop system satisfactory, except that the velocity constant $K_v = 14.1$ is lower than desired ($K_v = 100$). How might we improve the response?

Look at Bode plot:



Placing K_v constrain on Bode plot shows that we must somehow make slope steeper for a bit to achieve the requirement, if we want crossover behavior to be similar. We do this with a *lag* compensator:

$$\frac{s+a}{s+b}$$



On order to achieve our design goals, we need the lag ratio a/b to be the amount of additional low frequency gain required. In our case,

$$\frac{a}{b} = \frac{100}{14.1} = 7.1$$

We also need

$$a \ll \omega_c$$

So that not too much phase lag is added at crossover. It's common to use

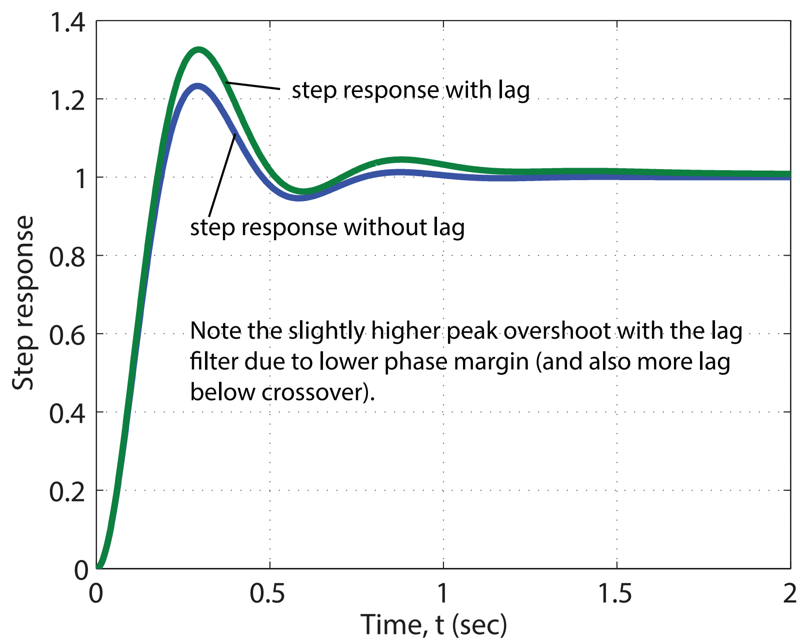
$$a = \omega_c/10$$

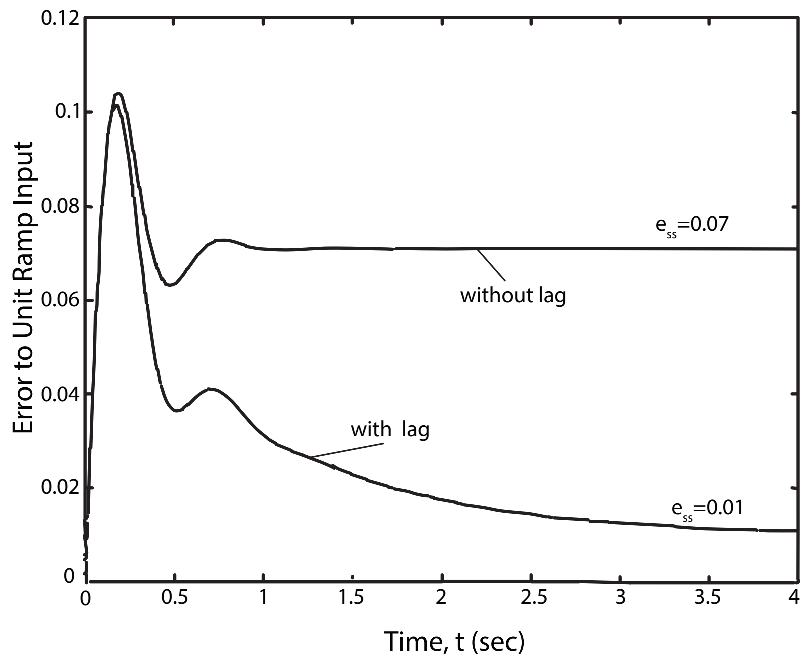
which ensures $< 6^\circ$ of phase lag will be added at crossover.

So the new Compensator is

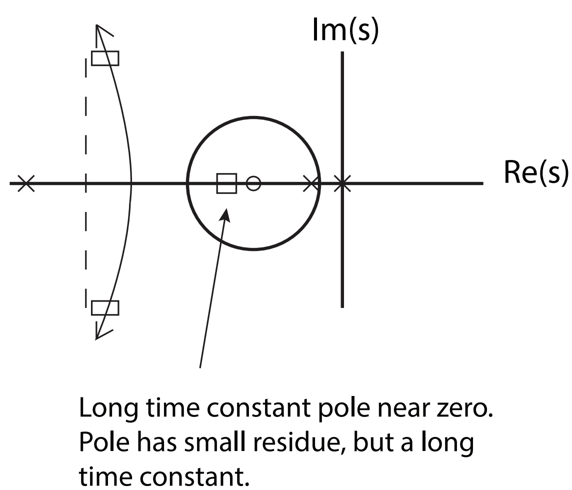
$$K(s) = 141\,\frac{s+1}{s+0.14}$$

How well does the new compensator work? Compare step responses, error response to ramp inputs (see plots).





Note that although the steady-state error to a ramp input is reduced, there is a long tail to the response. Why? Look at Root locus:



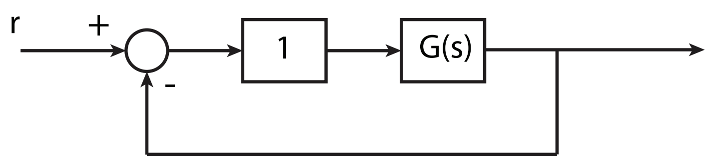
Note the constant pole near lag zero. Pole has small residue, but a long time constant.

This behavior is very typical of systems with lag of PI control. To eliminate, must increase bandwidth (crossover frequency), which is not always desirable.

PI Control

PI (proportional-integral control) is used when the type of the system must be increased, say, from type 0 to type 1.

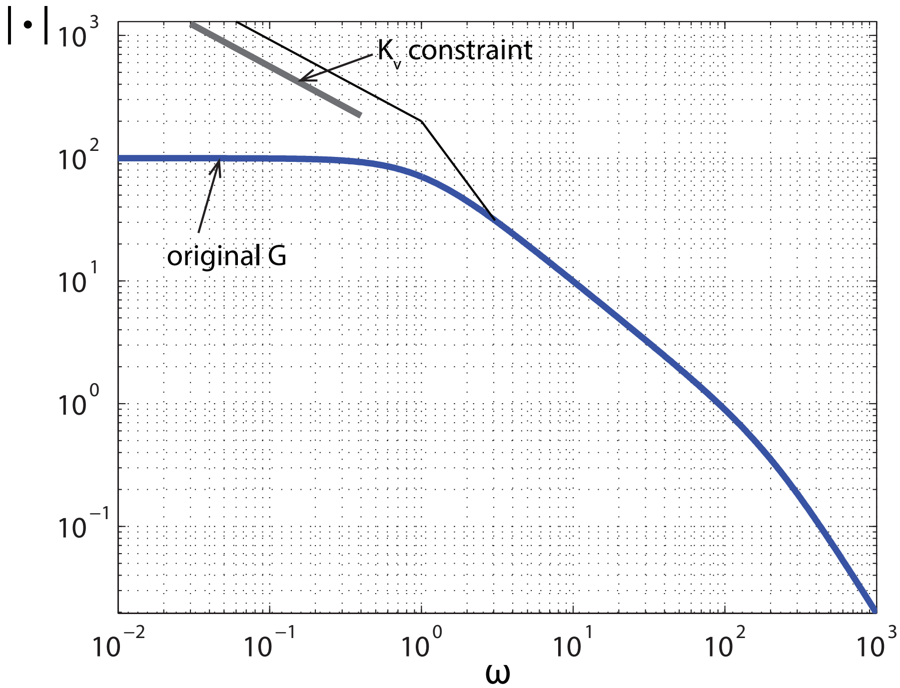
Example: Consider a system that performs adequately with unity feedback



where

$$G(s) = 100\,\frac{1}{(1 + s/1)(1 + s/200)},$$

but we desire a type 1 system with velocity constant $K_v = 100$. Look at problem on Bode plot:



So the compensator is

$$K(s) = \frac{3}{s} + 1 = \frac{s+3}{s}$$

Note that error pole will be near $s = -3$. To speed up error response, use

$$K(s) = \frac{s+10}{s} \qquad (\Rightarrow K_v = 1000)$$

which will result in pole near $s = -10$.