16.06 Principles of Automatic Control
Lecture 24

Compensation

Compensation is the use of a dynamic controller $K(s)$ (as opposed to proportional control) to improve the system's stability and error characteristics.

We have already seen compensation when we did root locus control, but we can do compensation more systematically using frequency response techniques.

We look primarily at four types of compensation:

PD Control Lead Compensation	used primarily to add lead at the crossover frequency, allowing the compensated system to have a faster speed of response and/or have more damping.
PI Control Lag Compensation	used primarily to increase the frequency response magnitude at low frequencies, reducing steady-state tracking errors.

Example:

Control the plant

$$G(s) = \frac{1}{s^2}$$

so that the rise time of the step response is

$$t_r \leqslant 1\,\text{sec}$$

and the overshoot is

$$M_p \leqslant 10\%$$

Solution:

We must first translate this requirements into frequency response characteristics. For a second order system,

$$
\begin{aligned}
M_p &= e^{-\pi \tan\theta} \\
\text{where } \theta &= \sin^{-1}\zeta
\end{aligned}
$$

So effective ζ required is

$$\zeta = 0.59(12)$$

Using the relationship

$$\zeta \approx \frac{\text{PM}}{100}$$

we see that the required phase margin is about 59°. So (rounding), require that PM = 60°.

Next, figure out what the crossover frequency is. By dimensional analysis, know that

$$t_r \approx \frac{1}{\omega_c}$$

Generally, will have

$$\underbrace{1/\omega_c}_{\text{for PM}\approx 0^\circ} \leqslant t_r \leqslant \underbrace{2.2/\omega_c}_{\text{for PM}\approx 90^\circ}$$

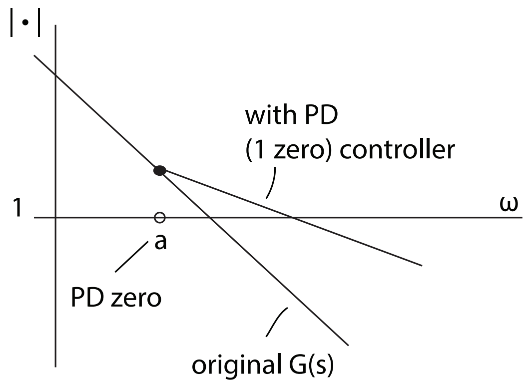
For other, more reasonable values of PM,

PM		$\omega_c t_r$
30	$\approx$	1.2
45	$\approx$	1.26
60	$\approx$	1.3

So let's try

$$
\begin{aligned}
\omega_c &= \frac{1.3}{t_r} = \frac{1.3}{1\,\text{sec}} \\
&= 1.3\,\text{r/s}
\end{aligned}
$$

What must loop look like?



To get PD correct, need 60° phase lead from PD zero,

$$K(s) = 1 + s/a$$

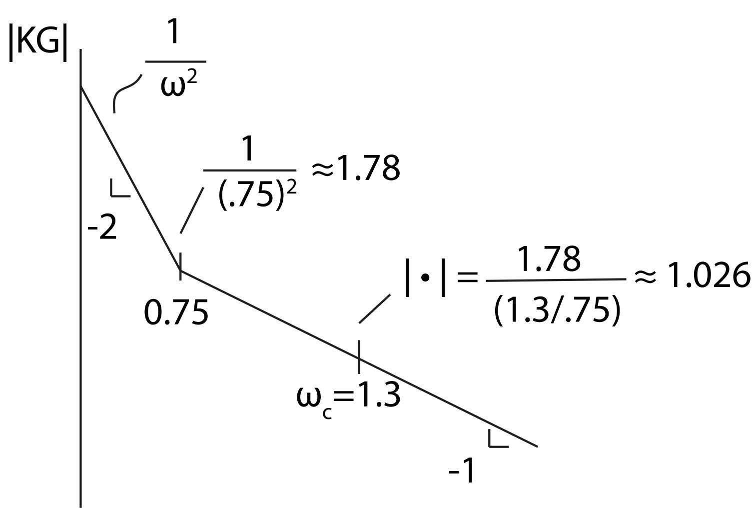
Since phase of $G(s)$ is $-180°$ everywhere, the phase is

$$\angle K(j\omega) = \tan^{-1}\omega/a.$$

To get 60°at ω_c, need

$$\tan^{-1}\frac{\omega_c}{a} = 60°$$
$$\Rightarrow a = \frac{\omega_c}{\tan 60°} = 0.75$$

If we use unit gain, what is $|KG|$ at crossover? Using straight lines,



Using exact expressions,

$$
\begin{aligned}
|KG(j\omega_c)| =& \frac{1}{\omega_c^2} \cdot \sqrt{1 + \omega_c^2/a^2} \\
=& 1.18
\end{aligned}
$$

So controller is

$$
\begin{aligned}
K(s) =& \frac{1}{1.18} \cdot (1 + s/0.75) \\
& \text{(PD controller)}
\end{aligned}
$$

Check results: Using Matlab, found that:

$$
\begin{aligned}
t_r =& 0.96 \quad \text{sec,} \quad \text{good!} \\
M_p =& 0.24, \quad \text{not to spec.}
\end{aligned}
$$

Part of the problem is that phase lag increases below crossover, increasing M_r, and therefore the peak overshoot. This will hopefully become a bit clearer when we do the Nichols plot. Could fix by increasing PM to $\approx 80^\circ$.

One problem with PD controller is that the gain is infinite at high frequencies. So instead use lead compensator

$$
K(s) = k \frac{1 + s/a}{1 + s/b}
$$

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