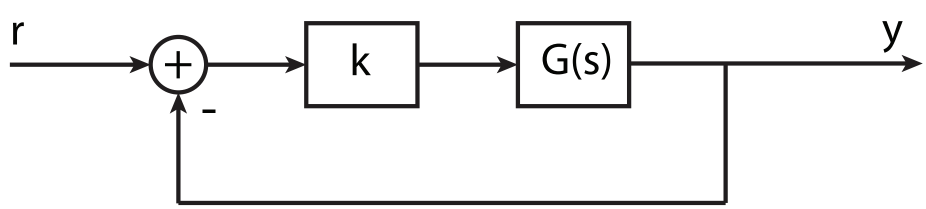
16.06 Principles of Automatic Control
Lecture 21

The Nyquist Stability Criterion

Can apply the argument principle to finding the stability of the closed loop system



The closed loop transfer function is

$$T(s) = \frac{Y(s)}{R} = \frac{kG(s)}{1 + kG(s)}$$

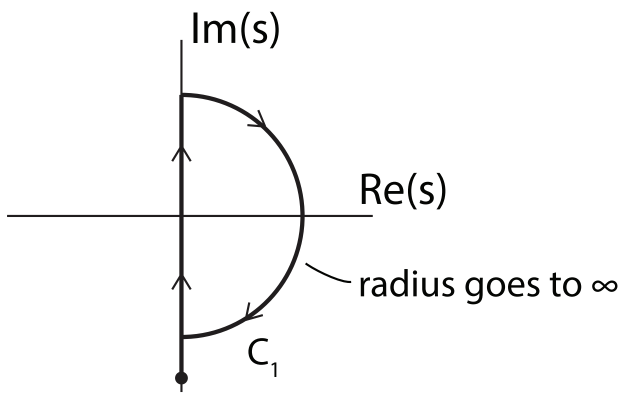
The closed loop poles of $T(s)$ are the roots of

$$0 = 1 + kG(s)$$

That is, the closed loop poles of $T(s)$ are zeros of $0 = 1 + kG(s)$. Note that the poles of $1 + kG(s)$ are just the open loop poles of $G(s)$. This suggests the following test for stability of the closed loop system:

Stability Test, Version1:

Define the contour C_1 as shown below:



The contour encloses (in the limit) the entire right half plane. For this contour, plot the contour map

$$1 + kG(s)$$

The number of CW encirclements of the origin by $1 + kG(C_1)$ is equal to $Z - P$, where Z is the number of closed loop poles in the right half plane, and P is the number of open loop poles in the right half plane. As an equation

$$Z = N + P$$

where Z - the number of closed loop unstable poles,
N - the number of CW encirclements of 0,
P - the number of unstable poles

Stability Test, Version 2:

Since the "1" term in $1 + kG(s)$ just shifts the contour map of $kG(s)$ by one unit to the right, it is often (usually) easier to plot $kG(s)$ alone. This is known as the *polar plot* or *Nyquist* plot for the system. Note that for each encirclement of 0 by $1 + kG(s)$, there is one encirclement of -1 by $kG(s)$. So the Nyquist Criterion, in the usual form, is

1. Plot $kG(s)$ for $-j\infty \leqslant s \leqslant j\infty$. First evaluate $kG(j\omega)$ for $\omega \in [0, \infty]$ and plot. Then reflect the image about the real axis and add to the previous image. Note that there no need to calculate $kG(s)$ on the circular part of C_1 if $kG(s) \to 0$ as $s \to \infty$.

2. Evaluate the number of CW encirclements about -1, and call that number N (see FPE for how to count encirclements).

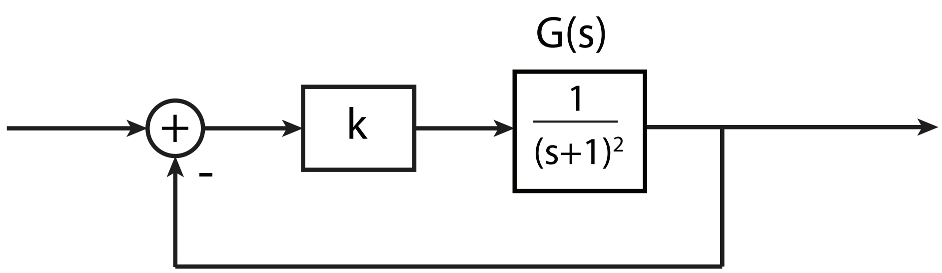
3. Determine the number of unstable poles of $G(s)$, P.

4. The number of unstable poles of the closed loop system is

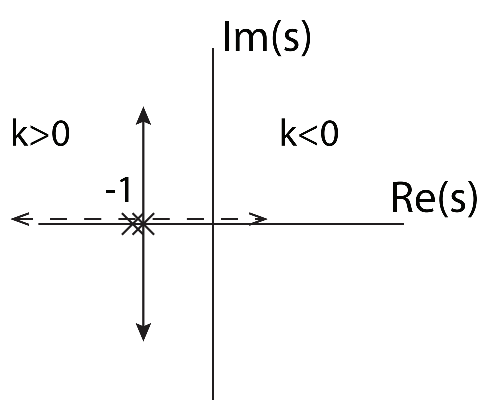
$$Z = N + P$$

Finally, if k is unknown, we can instead plot $G(s)$, and count encirclements of the point $-1/k$. This is useful for determining the range of gains for which the closed loop system is stable, as in root locus.

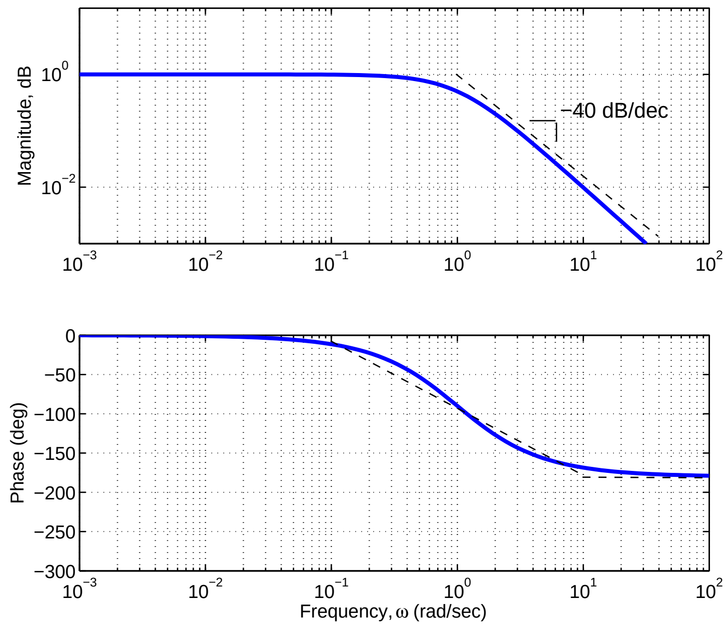
Examples



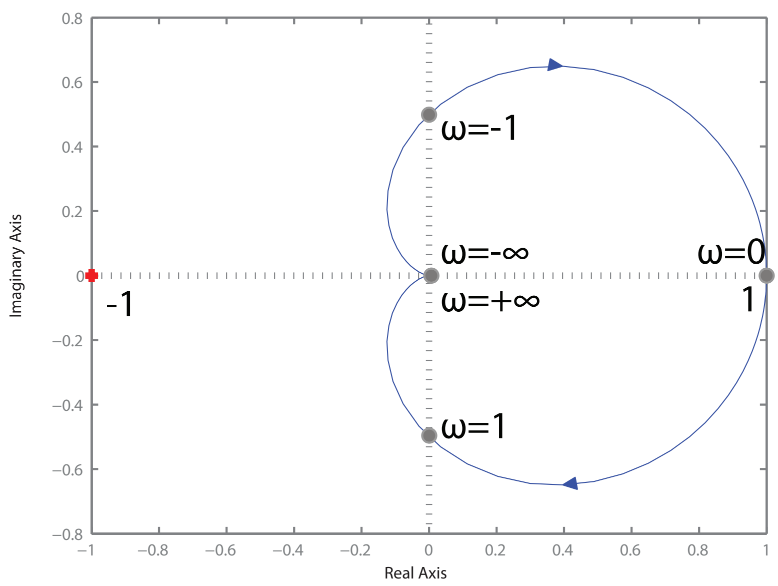
Root locus:



Bode plot:



Nyquist plot:



Note that the Nyquist plot does not encircle -1, and therefore the number of unstable closed loop poles is

$$
\begin{aligned}
Z =& N + P \\
=& 0 + 0 \text{ (no unstable open loop poles)} \\
=& 0, \text{ for } k = 1.
\end{aligned}
$$

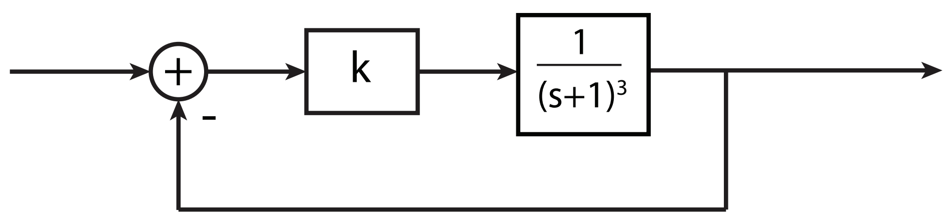
However, we can conclude more than that. The number of encirclements of $-1/k$ is zero for

$$
\begin{aligned}
-\frac{1}{k} &< 0 \quad \text{or} & -\frac{1}{k} &> 1 \\
\Rightarrow \frac{1}{k} &> 0 \quad \text{or} & \frac{1}{k} &< -1 \\
\Rightarrow 0 < k &< \infty \quad \text{or} & 0 > k &> -1
\end{aligned}
$$

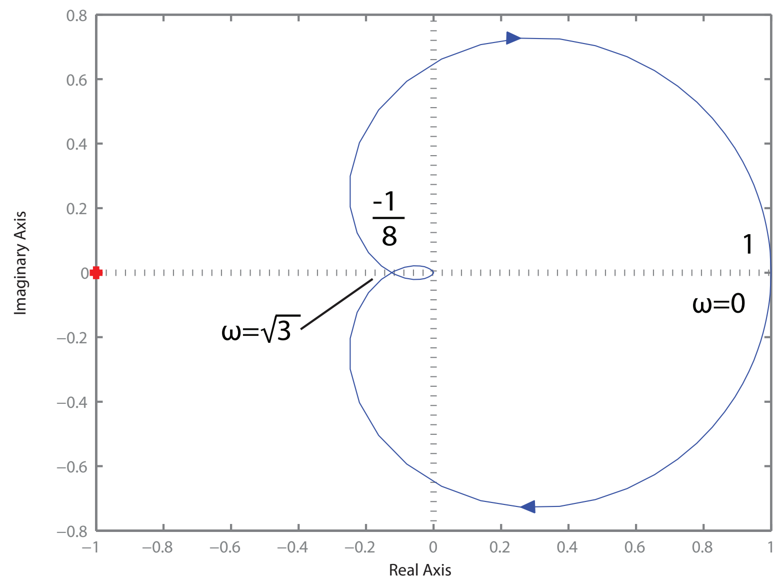
Therefore, the system is stable for $k > -1$.

For $k < -1$, $\quad N = 1$, so there is one unstable pole.

Example:



The Nyquist plot is:



For $-1/k < -1/8$ $(0 < k < 8)$, system is stable.

For $-1/k > 1$ $(0 > k > -1)$, system is stable.

For $-1/8 < -1/k < 0$, $\quad (k > 8)$, system has 2 unstable poles.

For $0 < -1/k < 1 \quad (k < -1)$, system has one unstable pole.

Of course, this agrees with our Routh and root locus analysis.