16.06 Principles of Automatic Control
Lecture 17

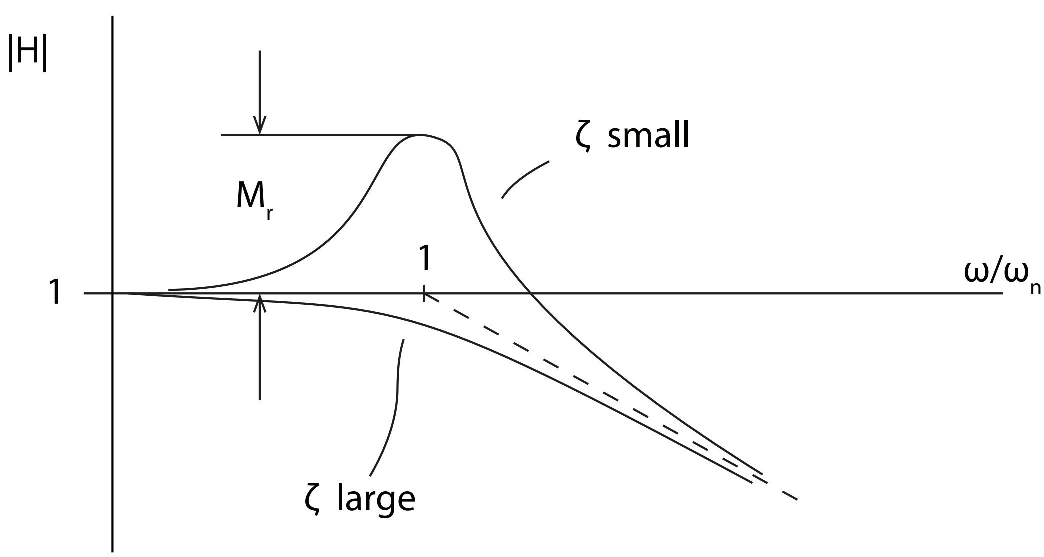
Frequency Response Design

Suppose we want to design a closed-loop system with a specific desired response. How might we use the FR to accomplish this?

$$\begin{aligned} H(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ &= \frac{1}{(\frac{s}{\omega_n})^2 + (\frac{2\zeta s}{\omega_n})^2 + 1} \quad \text{(Bode form)} \end{aligned}$$

The Bode (magnitude) plot is as shown in figure below.

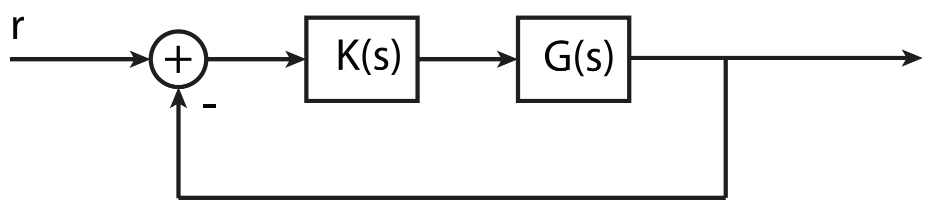
Note: See FPE Figure 6.3



We'll say more on Bode plot construction later. For now, a few importnat points:

- The magnitude of the resonant peak is M_r
- The resonant frequency, ω_r, is close to ω_n for lightly-damped systems with greater damping.
- The bandwidth (not shown) is the freq
- uency at which $\frac{|H(\omega)|}{|H(0)|} = 0.707$

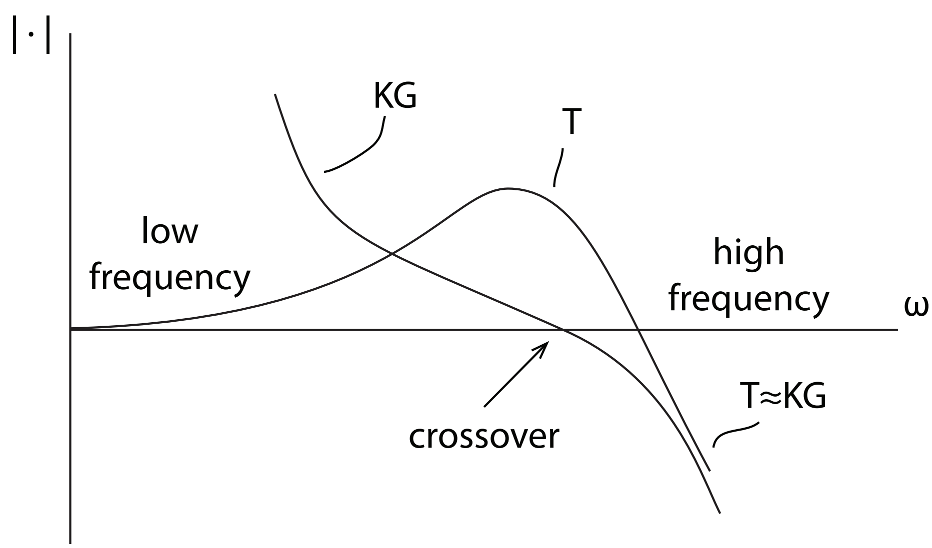
For a given unity-feedback control system



what will closed-loop transfer function

$$T(s) = \frac{K(s)G(s)}{1 + K(s)G(s)}$$

look like?



We can consider three regimes

1. Low frequency: $|KG| \gg 1$

   In this frequency range, $T(j\omega) \approx 1$.

2. High-frequency: $|KG| \ll 1$.

   In this frequency range, $T(j\omega) \approx K(j\omega)G(j\omega)$

3. Crossover: |KG|=1

   In this frequency range, $|T| = |\frac{KG}{1+KG}| = \frac{1}{|1+KG|}$

Note that at crossover, $|T|$ depends on the phase of KG.
If KG=1 (phase=0°), $|T| = 1/2$.
If KG=-1 (phase=$-180°$), $|T| = \infty$!
Bottom line is that the phase at crossover has a strong effect on $M_r$for the closed-loop system.
For now, it's enough to note that at crossover, we will have/want

$$-180^\circ < \angle KG < 0^\circ$$

In practice, we want the phase to be well away from $-180°$, but it will usually be less than $-90°$. A phase of $-120°$often works well, but that depends on the actual specifications.

Bode Plot Construction

The first step is to the transfer function of interest in Bode form

$$KG(s) = K_0 s^\alpha \frac{(1 + s/s_1)(1 + s/s_2)\dots}{(1 + s/s_a)(1 + s/s_b)\dots}$$

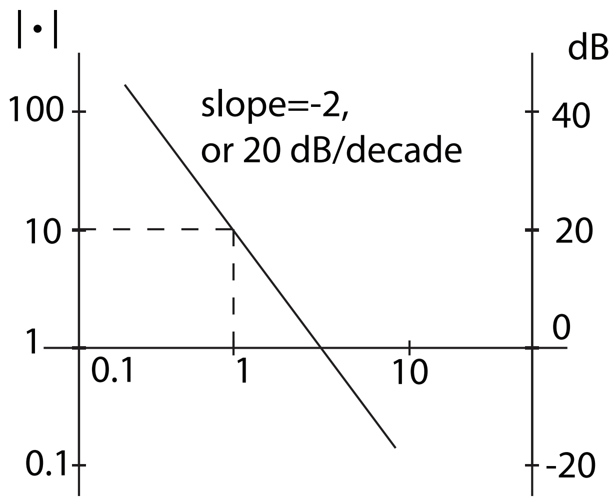
We can also have second order terms, which we will add later. Since we are plotting (for the magnitudinal plot), $\log|KG|$, we have

$$\begin{aligned}\log|KG(j\omega)| &= \log|K_0| + \alpha\log|\omega| \\ &+ \log|1 + j\omega/s_1| + \dots \\ &- \log|1 + j\omega/s_a| - \dots\end{aligned}$$

So on a log scale, plots of the individual terms *add*, since the log of a product is the sum of logs.
The $K_0 s^\alpha$ term is plotted as a straight line, since

$$\underbrace{\log|K_0(j\omega)^\alpha}_{\text{y-axis}} = \underbrace{\log K_0}_{\text{const}} + \underbrace{\alpha}_{\text{slope}} \underbrace{\log\ \omega}_{\text{x-axis}}$$

For example, plot magnitude of $10/s^2$:



To plot $1 + s/a$ term, note that

$$
\begin{aligned}
|1 + j\omega/a| &= (1 + \omega^2/a^2)^{1/2} \\
&= \begin{cases} 1 & \omega \ll a \\ \omega/a & \omega \gg a \\ \sqrt{2} & \omega = a \end{cases}
\end{aligned}
$$