16.06 Principles of Automatic Control
Lecture 14

Lead Compensator Example (cont-d)

To satisfy the angle condition, require that

$$\phi = 90 - 108.4 - 123.7 + 180(\mathrm{mod}\ 360) = 37.9^\circ$$

From the geometry shown,

$$tan\ \phi = \tfrac{3}{\beta - 3} \Rightarrow \beta = 6.9$$

To find the gain K, we must invoke the magnitude condition that

$$|K(s)G(s)| = 1$$

at the poles.

$$|K(s)G(s)| = 2 \cdot K \cdot \left| \frac{s+3}{(s+1)(s+2)(s+6\cdot 9)} \right| = 1$$

Therefore, $k = \frac{1}{2} \cdot \frac{3.61 \cdot 3.16 \cdot 4.92}{3} = 9.35$.
So choose compensator

$$K(s) = 9.35 \frac{s+3}{s+6.9}$$

The closed loop transfer function is

$$T(s) = \frac{Y}{R}(s) = 18.7 \frac{s+3}{s^3 + 9.9s^2 + 41.4s + 69.9}$$

The closed loop poles are at

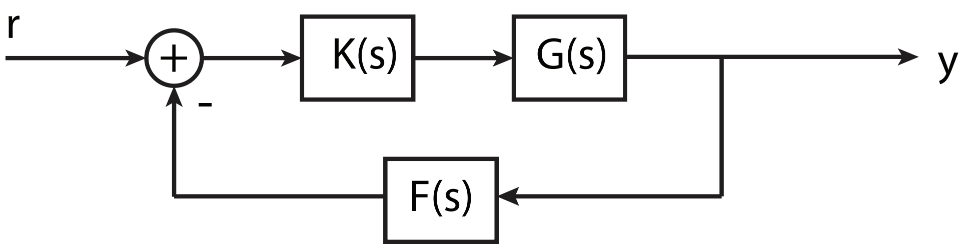
$$s = -3.015 \pm 2.996j, -3.87$$

The performance stats are:

$$
\begin{aligned}
M_p =& 8.4\% \leqslant 10\% \\
t_r =& 0.42 \text{ sec } \leqslant 0.5 \text{ sec} \\
k_p =& 4.06 \leftarrow \text{low}
\end{aligned}
$$

A note on Closed-Loop poles and zeros

Consider a closed-loop system with only one forward path and only one loop:



The poles of

$$T(s) = \frac{K(s)G(s)}{1 + K(s)G(s)F(s)}$$

can be found using root locus or other means.

The zeros of T(s) can be found by plugging in for K,G,F, and clearing fractions. However, the result is easy to state:

The zeros of T(s) are the zeros in the forwards path plus the poles in the feedback path. Suppose we add the requirement that

$$k_p \geqslant 20$$

So that steady-state tracking performance is acceptable. How can we modify the controller?

Lag Compensation

A lag compensator has the form

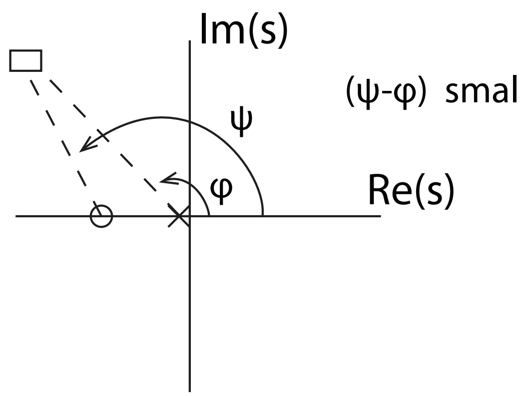
$$\frac{s + \alpha}{s + \beta}$$

where $\beta < \alpha$. Typically, α and β are much less (say, a factor of 100 than the natural frequency of the dominant poles.

Why does this work? At low frequency ($s \approx 0$), the gain of the lag compensator is

$$\frac{0+\alpha}{0+\beta} = \frac{\alpha}{\beta}$$

So the lag compensator increases the d.c. gain by the *lag ratio*. However, the effect on the dominant pole location is small:



The change in phase angle at the desired pole location is small ($\theta(5°)$), so the locus (away from the origin) doesn't change much.

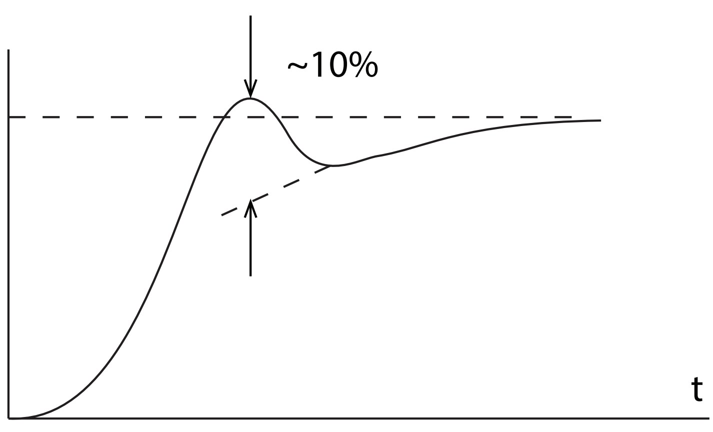
Continuing the example from last time, the natural frequency of the dominant pole is 4.24 rad/sec. Need lag ratio of 5. So use compensator

$$\frac{s+0.5}{s+0.1}$$

So compensator with both lead and lag is

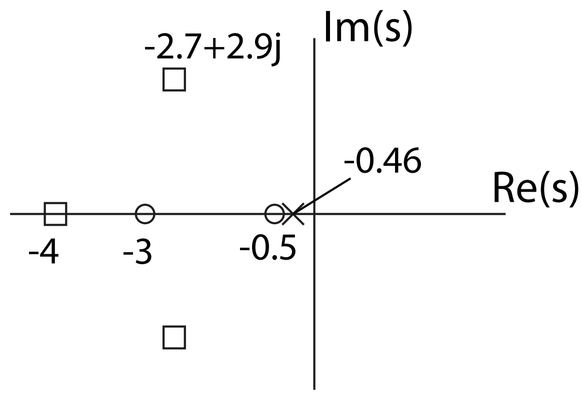
$$K(s) = 9.35\frac{s+3}{s+6.9} \cdot \frac{s+0.5}{s+0.1}$$

The response looks like:



The peak overshoot is 3%, but the "natural" overshoot is about 10%. There is also a slow, exponential tail. This is *typical* with lag compensation. Why?

The closed-loop pole-zero diagram is



The near pole-zero cancellation of pole at -0.46 means the effect of that pole is small, but it has a *very long time constant.* This pole is responsible for the small but long time constant error.

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