16.06 Principles of Automatic Control
Lecture 10

PID Control

A common way to design a control system is to use PID control.

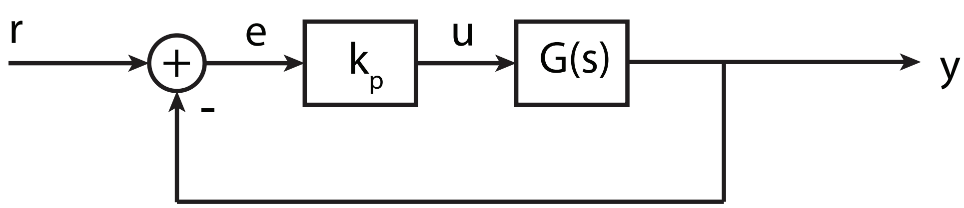
PID = proportional-integral-derivative

Will consider each in turn, using an example transfer function

$$G(s) = \frac{A}{s^2 + a_1 s + a_2}$$

Proportional (P) control

In proportional control, the control aw is simply a gain, to that u is proportional to e:



$$u = k_p e$$

For our example, the characteristic equation is

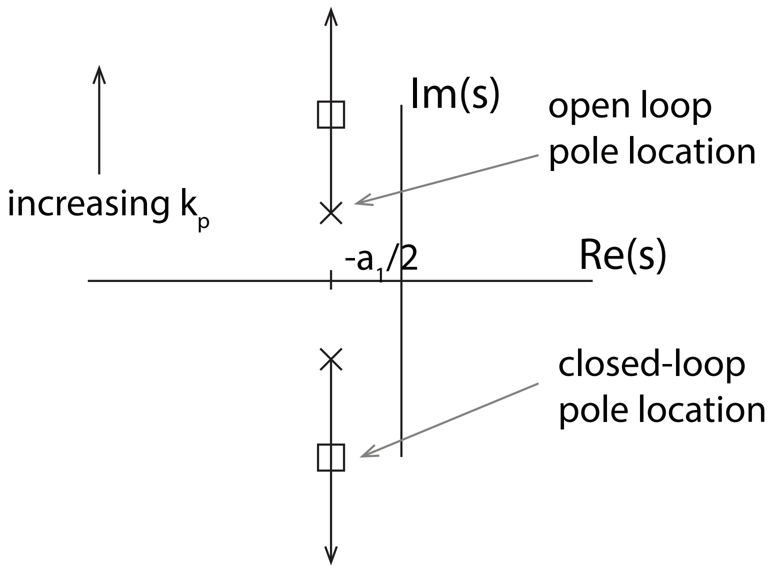
$$\begin{aligned} 0 =& 1 + k_p G(s) \\ =& 1 + \frac{k_p A}{s^2 + a_1 s + a_2} \\ \Rightarrow 0 =& s^2 + a_1 s + a_2 + k_p A \end{aligned}$$

The resulting natural frequency is

$$\omega_n = \sqrt{a_2 + k_p A}$$

So in the example, increasing k_p increases the natural frequency, but reduces the damping ratio.

Plot of pole location vs k_p:



Derivative (D) control

To add damping to a system, it is often useful to add a derivative term to the control,

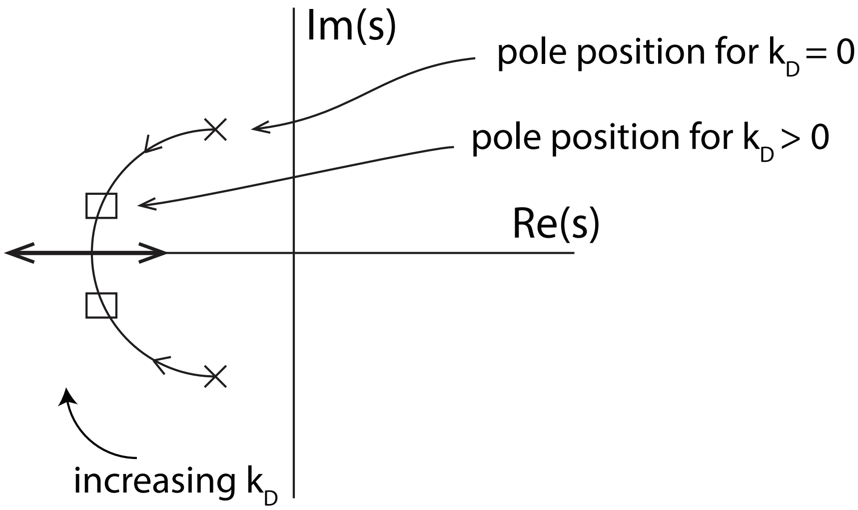
$$\begin{aligned}
u(t) =& k_p e(t) + k_D \dot{e}(t) \\
& \text{or} \\
v(s) =& k_p E(s) + k_D s E(s) \\
=& (k_p + k_D s) E(s) \\
=& K(s) E(s)
\end{aligned}$$

What is the characteristic equation?

$$\begin{aligned}
0 =& 1 + K(s)G(s) \\
=& 1 + \frac{(k_p + k_D s)A}{s^2 + a_1 s + a_2} \\
0 =& s^2 + (a_1 + k_D A)s + (a_2 + k_p)
\end{aligned}$$

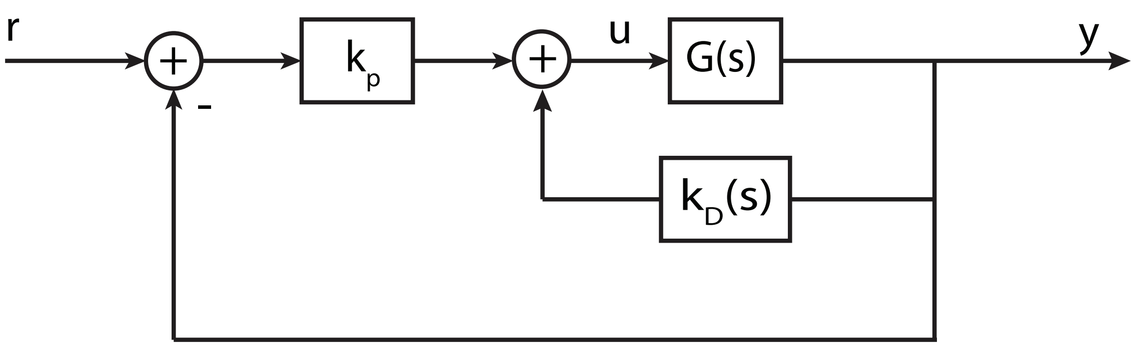
So increasing k_D increases the damping ratio without changing the natural frequency, *for this example.*

For $k_p$fixed, $k_D$varying, plot of closed-loop pole location is:



NB: For other $G(s)$, results may vary.

Sometimes, it's better to place derivative feedback in the feedback path:



Why? We get the same pole locations, but no additional zeros to cause additional overshoot. Another way to think about this is that we want the derivative effect on y, because that adds damping, but we don't want to differentiate the reference.

Integral (I) control

Especially if the plant is a type 0 system, we may want to add integrator to controller to drive steady-state error to zero:

$$V(s) = \underset{\substack{\downarrow \\ P}}{(k_p} \quad \underset{\substack{\downarrow \\ I}}{+\frac{k_I}{s}} \quad \underset{\substack{\downarrow \\ D}}{+k_D s)} \quad E(s)$$

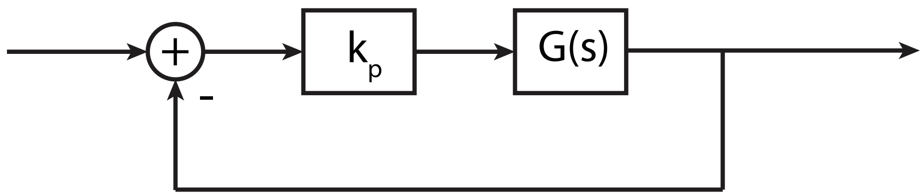
Example:

$$G(s) = \frac{1}{s^2 + s + 1}$$

Suppose we want a system that

1. Has rise time above $t_r = 1$s
2. Has peak overshoot of $M_p = 0.05$
3. Has zero steady-state error to step command

Let's do one piece at a time:



Characteristic equation is

$$0 = s^2 + s + 1 + k_p$$

So can only change ω_n (and indirectly, ζ) with k_p. for $t_r = 1$, need

$$1 = \frac{1.8}{\omega_n} \Rightarrow \omega_n \approx 1.8$$

So let's take $k_p = 2$ for simplicity. Then

$$T = \frac{k_p G}{1 + k_p G} = \frac{3}{s^2 + s + 4}$$
$$\Rightarrow \zeta = 0.25, \quad \text{Low}$$

To get $M_p = 5\%$, need $\zeta = 0.7$. So add derivative control. Characteristic Equation is

$$0 = s^2 + (1 + k_p)s + 1 + k_p$$

The desired polynomial is

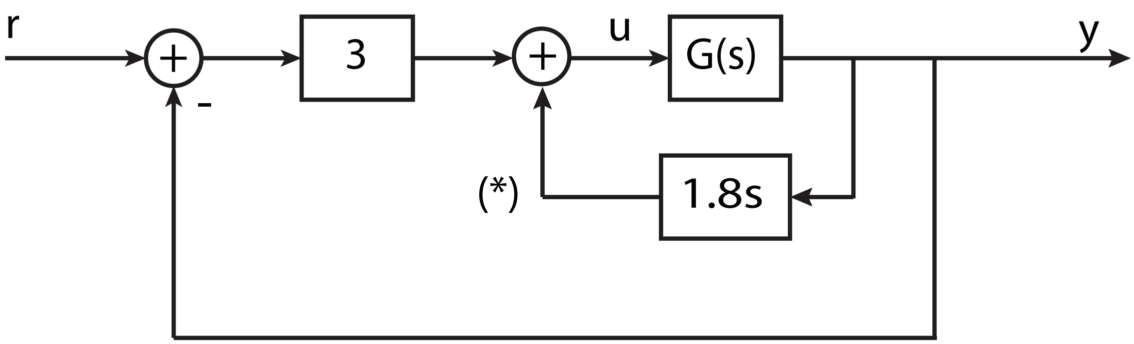
$$0 = s^2 + 2.8s + 4$$

So take $k_D = 1.8$.

If PD control is in forward loop,

$$T = \frac{1.8s + 3}{s^2 + 2.8s + 4}$$

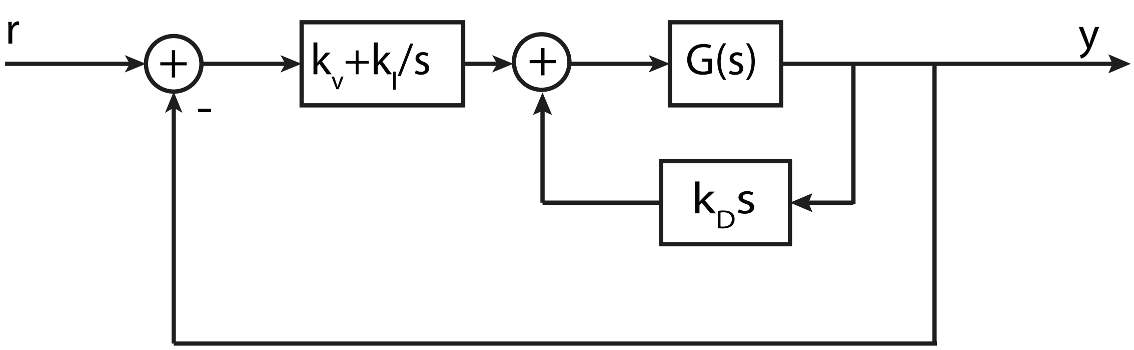
and the peak overshoot will be 16%, not 5%. So instead, use control structure



(∗) = ”mirror loop feedback”
With this structure, we have:

$$\begin{aligned} t_r &= 1.06s \\ M_p &= 4.6\% \\ e_{ss} &= 0.25 \end{aligned}$$

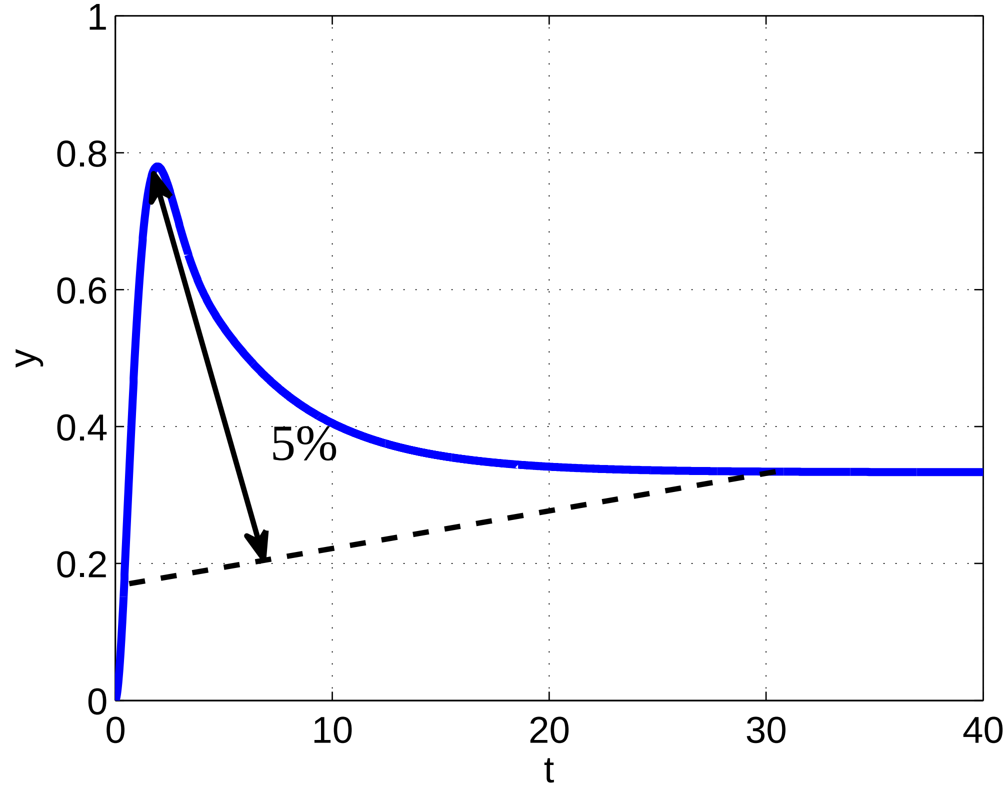
So let’s add integral control:



Take $k_I = 0.25$ (trust me!)
Then

$$T = \frac{3s + 0.25}{s^3 + 2.8s^2 + 4s + 0.25}$$

Response *sort of* meets specs:



The response has a long tail, due to slow pole – poles are at:

$$s = -1.37 \pm 1.40j$$
$$s = -0.065$$

↑ slow pole causes long tail