

Solution to U1 by Waitz. (Range Equation)

a) Assuming steady-level flight and no fuel reserves, estimate the range of a B-777 using the information given in the lecture notes (and/or on Boeing's web page). How well does this compare to the estimates Boeing publishes on their web page?

BASIS FOR COMPARISON: BOEING 777-200/200IGW

Max. take-off mass $\approx 275,000$ kg
Typ. operating empty mass $\approx 144,000$ kg
Max. fuel capacity $\approx 171,000$ liters (137,000 kg kerosine)
Cargo volume $\approx 160,000$ liters
Takeoff thrust ≈ 760 kN
Cruise thrust ≈ 160 kN
Design range ≈ 9600 to 13800 km
Passengers ≈ 330
Length ≈ 63.7 m, wingspan = 60.9m
L/D ≈ 18 , $\eta_o \approx 0.36$
Cost $\approx \$140$ million

Using:

$$\text{Range} = \frac{h}{g} \frac{L}{D} \eta_{\text{overall}} \ln \frac{W_{\text{initial}}}{W_{\text{final}}}$$

You get about 18000km using the ratio of the operating empty mass and the max takeoff mass (1.9). The estimate of 18000km is more than 30% too high, but I did neglect the weight of the passengers and their cargo, food (such as it is), and reserve fuel. When these items are taken into account the estimate is within 10% of the published values.

b) Now assuming that L/D, propulsion system efficiency and final weight are unchanged, estimate the range of a B-777 if the same volume of liquid hydrogen were to be used instead of Jet-A.

To do this I wrote $W_{\text{final}} = W_{\text{initial}} - W_{\text{fuel}} = W_{\text{initial}} - \rho_{\text{fuel}} V_{\text{fuel}}$. The ratio of the two densities is 0.0875. So the initial weight is only 156,000 kg (144,000kg + 0.0875x137,000kg), and the weight ratio drops to 1.08. Of course the heating value is increased by a factor of 2.8, but it hardly makes up for the reduction in the amount of energy that is carried due to hydrogen's low density. My estimate for the range is 6100km, a reduction by a factor of three from the case with Jet-A.

c) Derive an equation for the range of a battery-powered aircraft in steady-level flight. Express the range in terms of L/D, propulsion system efficiency, battery mass and heating value, and aircraft weight. Estimate the range of a B-777 if the fuel was taken out and replaced with its equivalent weight in batteries.

The key with a battery-powered aircraft is that its mass does not change as it burns the energy. This makes the range equation more straightforward.

$m_b \cdot h = \text{energy available in the battery (J)}$

$\frac{T \cdot u_o}{\eta_{overall}} = \text{rate of energy usage to overcome drag (J/s)}$

$$\text{time of flight} = \frac{m_b \cdot h}{\frac{T \cdot u_o}{\eta_{overall}}} \quad (s)$$

$$\text{Range of flight} = u_o \left(\frac{m_b \cdot h}{\frac{T \cdot u_o}{\eta_{overall}}} \right) \quad (m)$$

or

$$\text{Range of flight} = \frac{m_b \cdot h \cdot \eta_{overall}}{T} = \frac{m_b \cdot h \cdot \eta_{overall}}{W} \left(\frac{L}{D} \right) \quad (m)$$

With $m_b = 137,000\text{kg}$, $h=2.5\text{MJ/kg}$, $W=(275,000\text{kg})(9.8\text{m/s}^2)=2695\text{kN}$, I calculate the range to be: 820km. As you can see, the low energy density of the battery is a disaster for range—it is reduced by a factor of more than 20 relative to the Jet-A powered model.

“FUEL”	Heating Value (MJ/kg)	Density (kg/m ³)
Jet-A	42.8	800
Liquid Hydrogen	120	70
Batteries	2.5	8000