

# COMPENDIUM OF EQUATIONS

## Unified Engineering Thermodynamics

Note: *It is with some reservation that I supply this compendium of equations. One of the common pitfalls for engineering students is that they solve problems through pattern matching rather than through applying the correct equation based upon a foundation of conceptual understanding. This type of training does not serve the students well when they are asked to perform higher-level cognitive tasks such as analysis, synthesis, and evaluation. Thus, while you are welcome to use this list as a guide and a study aid, I expect you to be able to derive each of these equations from their most general forms (e.g. work, the First Law, etc.). Do not settle for a shallow understanding of this material.*

### I. Equation of State:

$$pv = RT \quad \text{or} \quad p = \frac{RT}{v} \quad \text{for a thermally perfect gas}$$

### II. Expressions for Work:

A. Work for a simple compressible substance

$$W = \int_{v_1}^{v_2} p_{ext} dV$$

B. Work for a simple compressible substance undergoing a quasi-static process

$$W = \int_{v_1}^{v_2} p dV$$

C. Work for an isothermal, quasi-static process of a simple compressible substance

$$W = mRT \ln \frac{v_2}{v_1} = mRT \ln \frac{p_1}{p_2}$$

D. Work for an isobaric quasi-static process of a simple compressible substance

$$W = p(V_2 - V_1)$$

E. Work for a quasi-static adiabatic process

$$W = -(U_2 - U_1)$$

F. Work for quasi-static adiabatic process of an ideal gas

$$W = -mc_v(T_2 - T_1)$$

### III. Forms of the First Law of Thermodynamics

A. Most general forms

$$E = Q - W, \quad e = q - w, \quad dE = Q - W, \quad \text{and} \quad de = q - w$$

B. Neglecting changes in kinetic and potential energy

$$U = Q - W \quad u = q - w, \quad dU = Q - W, \quad \text{and} \quad du = q - w$$

C. Neglecting changes in kinetic and potential energy, in terms of enthalpy

$$H = U + pV \quad \text{therefore} \quad dH = dU + pdV + Vdp$$

$$\text{so} \quad dH = Q - W + pdV + Vdp$$

$$\text{or} \quad dh = q - w + pdv + vdp$$

D. For quasi-static processes where changes in kinetic and potential energy are not important.

$$dU = Q - pdV \quad \text{or} \quad du = q - pdv$$

$$dH = Q + Vdp \quad \text{or} \quad dh = q + vdp$$

E. For quasi-static processes of an ideal gas where changes in kinetic and potential energy are not important.

$$mc_v dT = Q - pdV \quad \text{or} \quad c_v dT = q - pdv$$

$$mc_p dT = Q + Vdp \quad \text{or} \quad c_p dT = q + vdp$$

### IV. The First Law of Thermodynamics as a Rate Equation

A. Most general form

$$\frac{dE_{c.v.}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \dot{m}_{in}e_{in} - \dot{m}_{out}e_{out}$$

$$\begin{array}{l} \text{rate of change} \\ \text{of energy in c.v.} \end{array} = \begin{array}{l} \text{rate of heat} \\ \text{added to c.v.} \end{array} - \begin{array}{l} \text{rate of work} \\ \text{done} \end{array} + \begin{array}{l} \text{rate of energy} \\ \text{flow in to c.v.} \end{array} - \begin{array}{l} \text{rate of energy} \\ \text{flow out of c.v.} \end{array}$$

B. For a steady flow process

$$\frac{d}{dt} = 0 \quad \text{and} \quad \dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}$$

$$\dot{Q}_{\text{c.v.}} - \dot{W}_{\text{c.v.}} = \dot{m}(e_{\text{out}} - e_{\text{in}})$$

or

$$\dot{Q}_{\text{c.v.}} - \dot{W}_{\text{c.v.}} = \dot{m}[(\text{IE} + \text{KE} + \text{PE})_{\text{out}} - (\text{IE} + \text{KE} + \text{PE})_{\text{in}}]$$

C. For a steady flow process neglecting changes in potential energy

$$\dot{Q}_{\text{c.v.}} - \dot{W}_{\text{c.v.}} = \dot{m} \left[ u + \frac{c^2}{2} \right]_{\text{out}} - \left[ u + \frac{c^2}{2} \right]_{\text{in}}$$

or

$$q_{1-2} - w_{1-2} = u_2 - u_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

written in terms of external or shaft work

$$q_{1-2} - w_{s1-2} = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

or in terms of shaft work and enthalpy

$$q_{1-2} - w_{s1-2} = h_2 - h_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}$$

D. Steady flow energy equation for an ideal gas

$$q_{1-2} - w_{s1-2} = c_p T_2 + \frac{c_2^2}{2} - c_p T_1 + \frac{c_1^2}{2}$$

E. Steady flow energy equation for an ideal gas for an adiabatic process with no shaft work

$$c_p T_2 + \frac{c_2^2}{2} = c_p T_1 + \frac{c_1^2}{2}$$

The quantity that is conserved is called the stagnation temperature.

$$T_T = T + \frac{c^2}{2c_p} \quad \text{or} \quad \frac{T_T}{T} = 1 + \frac{-1}{2} M^2 \quad \text{using } a = \sqrt{RT}$$

It is also convenient to define the **stagnation enthalpy**,  $h_T$

$$h_T = c_p T + \frac{c^2}{2}$$

so we can rewrite the **Steady Flow Energy Equation** in a convenient form as

$$q_{1-2} - w_{s1-2} = h_{T2} - h_{T1}$$

F. Steady flow energy equation for an ideal gas for a quasi-static adiabatic process with no shaft work

$$\frac{p_T}{p} = 1 + \frac{-1}{2} M^2 \quad /^{-1}$$

## V. Other relationships

A. Relationship between properties for quasi-static, adiabatic processes for thermally perfect gases

$$pv = \text{constant}$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \quad /^{-1} \quad \text{and} \quad \frac{T_2}{T_1} = \frac{v_1}{v_2} \quad ^{-1} \quad \text{and} \quad \frac{P_2}{P_1} = \frac{v_1}{v_2}$$

B. Thermal efficiency of a cycle

$$= \frac{\text{net work}}{\text{heat input}} = \frac{w}{q_{\text{comb.}}}$$

C. Entropy

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

For the case of a thermally perfect gas then

$$s - s_0 = c_v \int_{T_0}^T \frac{dT}{T} + R \ln \frac{v}{v_0}$$

or in situations with  $c_v = \text{constant}$

$$s - s_0 = c_v \ln \frac{T}{T_0} + R \ln \frac{v}{v_0}$$

So for the case of a thermally perfect gas then

$$s - s_0 = c_p \frac{T}{T_0} - R \ln \frac{p}{p_0}$$

or in situations with  $c_p = \text{constant}$

$$s - s_0 = c_p \ln \frac{T}{T_0} - R \ln \frac{p}{p_0}$$

## VI. Nomenclature

a	speed of sound (m/s)
c	velocity (m/s)
$c_p$	specific heat at constant pressure (J/kg-K)
$c_v$	specific heat at constant volume (J/kg-K)
e	energy (J/kg)
E	energy (J)
h	enthalpy (J/kg)
$h_T$	total or stagnation enthalpy (J/kg)
H	enthalpy (J)
m	mass (kg)
p	pressure (kPa)
$p_T$	total or stagnation pressure (kPa)
q	heat (J/kg)
Q	heat (J)
R	gas constant (J/kg-K)
s	entropy (J/K)
S	entropy (J/kg-K)
t	time (s)
T	temperature (K)
$T_T$	total or stagnation temperature (K)
u	internal energy (J/kg)
U	internal energy (J)
v	specific volume ( $\text{m}^3/\text{kg}$ )
V	volume ( $\text{m}^3$ )
w	work (J/kg)
$w_s$	shaft or external work (J/kg)
W	work (J)
	ratio of specific heats, $c_p/c_v$
	thermal efficiency
	density ( $\text{kg}/\text{m}^3$ )