PRODUCTION OF THRUST

Newton's 2nd Law (*e.g.* Σ F = d/dt (mv))
for a control volume of fixed mass with steady flow in and out and no acceleration of the frame of reference relative to inertial coordinates:

$$\sum \overline{F} = \int_s \overline{u}(\rho \overline{u}) \cdot \overline{n} ds$$

Sum of external forces on control volume, e.g.

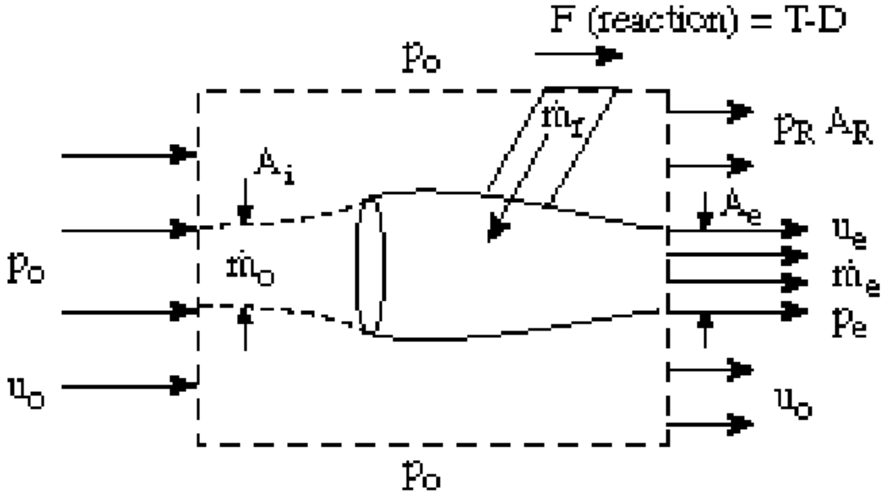
- Pressure forces
- Shear forces
- Body forces
- Reaction forces

Net flux of momentum through surface of control volume

For x-component of vectors:

$$\sum F_x = \int_s u_x \rho \overline{u} \cdot \overline{n} ds$$

PRODUCTION OF THRUST



(A_R = rest of area not taken by A_e)

$$T - D + \sum \text{Pressure Forces} = \int_s \rho u_x (\bar{u} \cdot \bar{n}) dA$$

$$T - D + \left[-P_e A_e + P_o A_e - \int_{A_R} (P_R - P_o) dA_R \right] = \int_s \rho u_x (\bar{u} \cdot \bar{n}) dA$$

$$\int_s \rho u_x \bar{u} \cdot \bar{n} dA = \rho_e u_e A_e u_e - \rho_o u_o A_o u_o + \int_{C_s - A_o - A_e} \rho u_x \bar{u} \cdot \bar{n} dA$$

$$= \dot{m}_e u_e - \dot{m}_o u_o + \int_{C_s - A_o - A_e} \rho u_x \bar{u} \cdot \bar{n} dA$$

$$T - D - (P_e - P_o) A_e - \int_{A_R} (P_R - P_o) dA_R = \dot{m}_e u_e - \dot{m}_o u_o + \int_{C_s - A_e - A_o} \rho u_x \bar{u} \cdot \bar{n} dA$$

PRODUCTION OF THRUST

Everything that relates to flow through the engine is conventionally called thrust. Everything that relates to the flow on the outside of the engine is conventionally call drag. Therefore, gathering only those terms that relate to the fluid that passes through the engine, we have:

$$T = \dot{m}_e u_e - \dot{m}_o u_o + (P_e - P_o) A_e$$

The thrust is largely composed of the net change in momentum of the air entering and leaving the engine, with a typically small adjustment for the differences in pressure between the inlet and the exit.

EFFICIENCY

We have related the thrust of a propulsion system to the net changes in momentum, pressure forces, etc. Now we will look at how efficiently the propulsion system converts one form of energy to another on its way to producing thrust:

$$\textit{overall efficiency} = \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{propulsive power}}{\text{fuel power}}$$

$$\textit{propulsive power} = \text{thrust} \cdot \text{flight velocity} = Tu_o$$

$$\textit{fuel power} = \text{fuel mass flow rate} \cdot \text{fuel energy per unit mass} = \dot{m}_f h$$

Thus,

$$\eta_{overall} = \frac{Tu_o}{\dot{m}_f h}$$

EFFICIENCY

It is often convenient to break the overall efficiency into: thermal efficiency and propulsive efficiency where

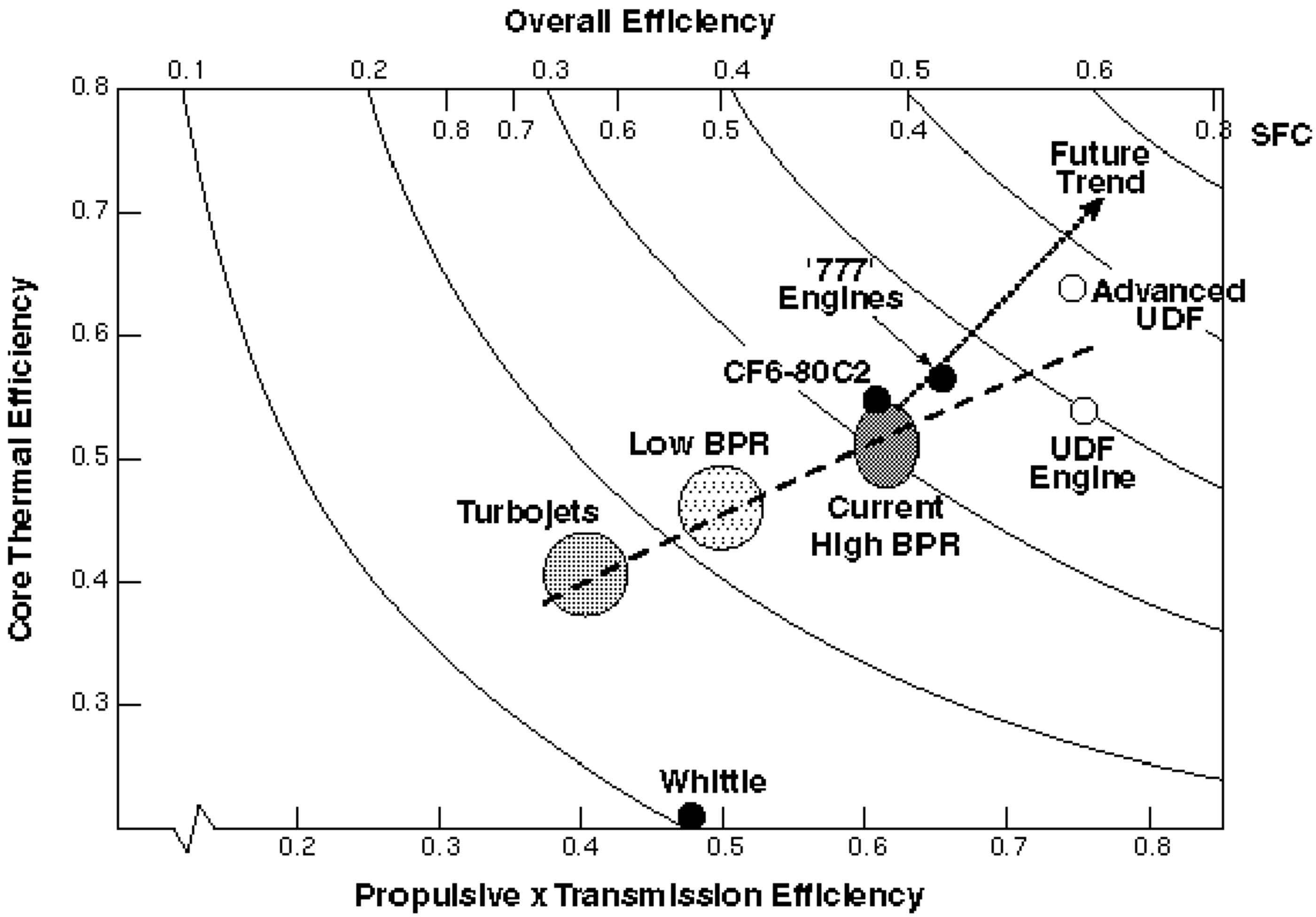
$$\eta_{thermal} = \frac{\text{rate of production of propellant k.e.}}{\text{fuel power}} = \frac{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_o u_o^2}{2}\right)}{\dot{m}_f h}$$

$$\eta_{prop} = \frac{\text{propulsive power}}{\text{rate of production of propellant k.e.}} = \frac{T u_o}{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_o u_o^2}{2}\right)}$$

Such that,

$$\eta_{overall} = \eta_{thermal}\eta_{propulsive}$$

OVERALL PROPULSION SYSTEM EFFICIENCY



Trends in *thermal efficiency* are driven by increasing compression ratios and corresponding increases in turbine inlet temperature. Whereas trends in *propulsive efficiency* are due to generally higher bypass ratio engines

(After Koff, 1991)

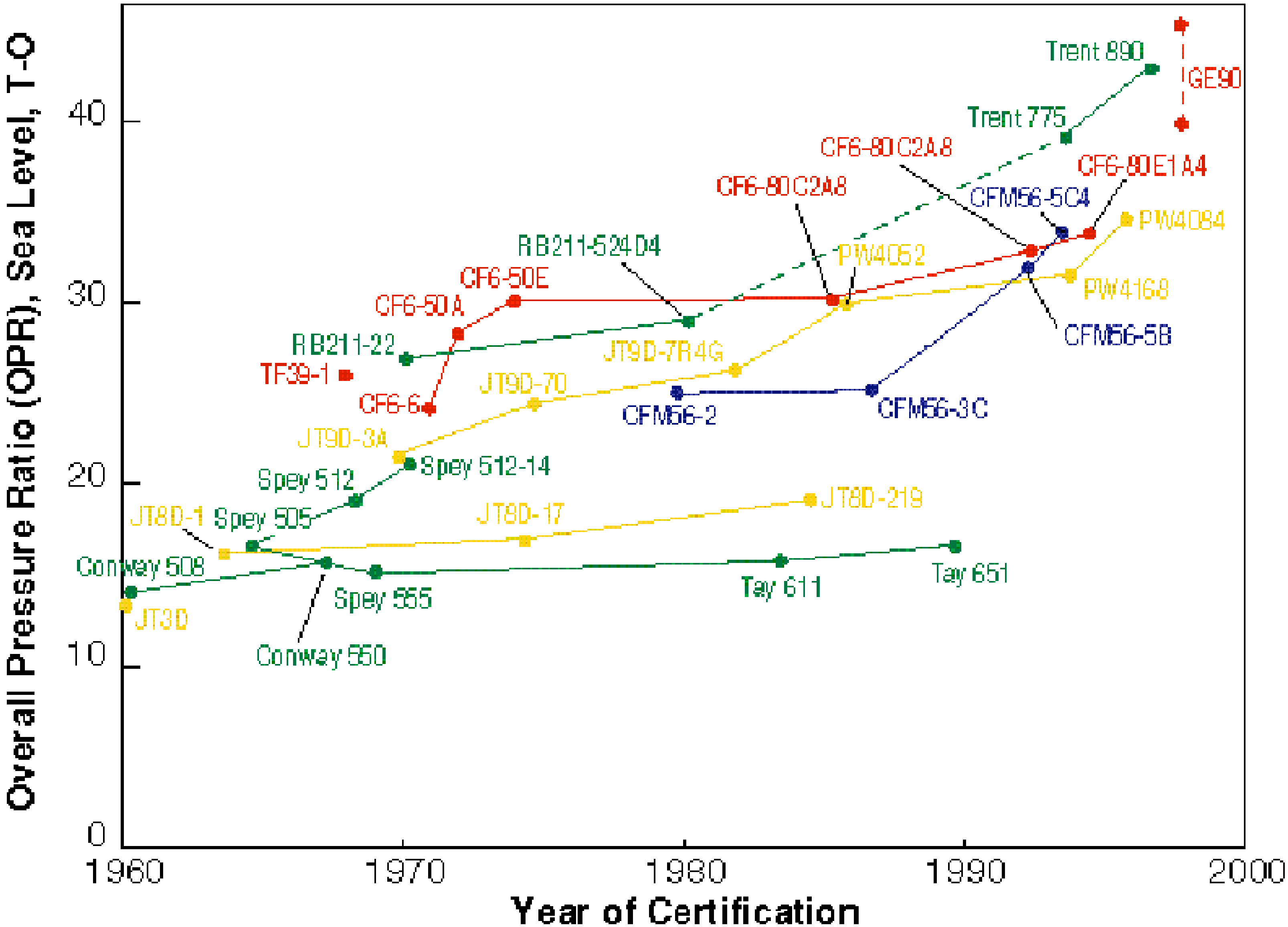
EFFICIENCY

The thermal efficiency is the same as that used in thermodynamics. For an ideal Brayton cycle it is a function of the temperature ratio across the compressor

$$\eta_{th-ideal\,Brayton\,cycle} = \frac{W_{net}}{Q_{in}} = 1 - \frac{T_{atm}}{T_{comp.\,exit}} = 1 - \frac{1}{(PR)^{\gamma-1/\gamma}}$$

Higher temperature ratio = higher pressure ratio = higher thermal efficiency

PRESSURE RATIO TRENDS



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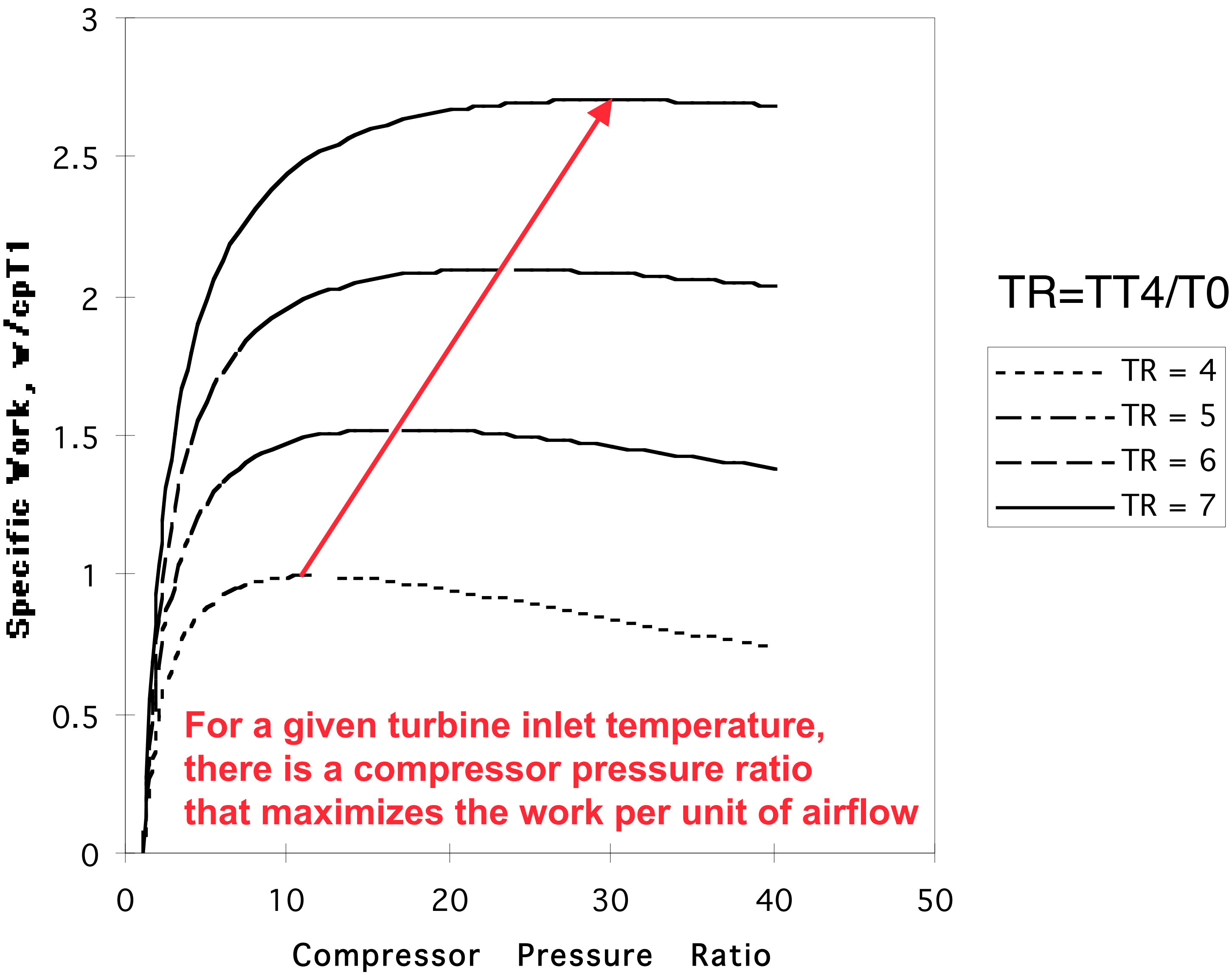
TURBINE INLET TEMPERATURE

- **Desire for higher turbine inlet temperature is driven by desire for high specific work**

- **Specific work is work per unit of airflow**

- **High work per unit of airflow = smaller engine, lower weight, etc.**

BRAYTON CYCLE SPECIFIC WORK



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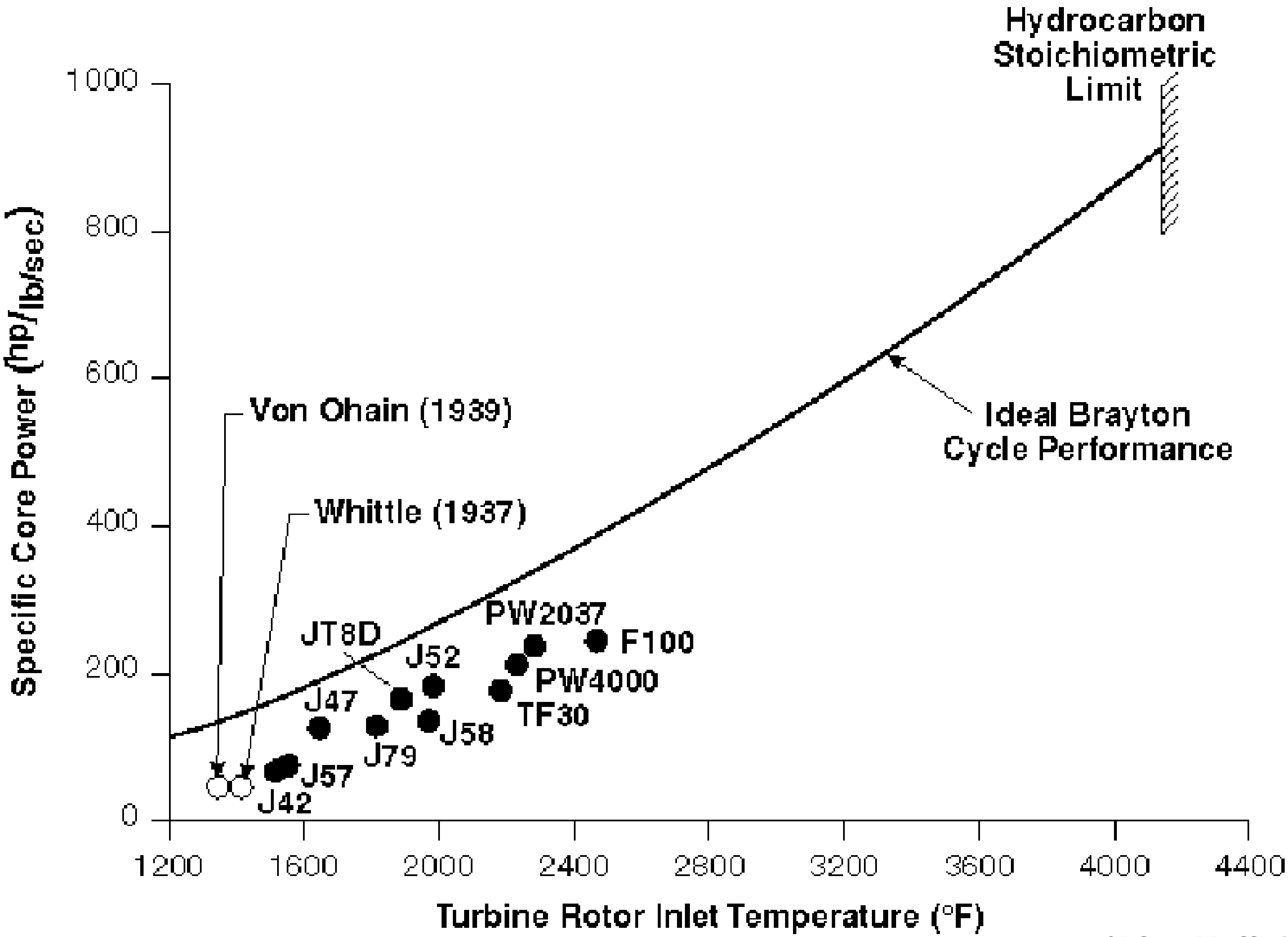
TURBINE INLET TEMPERATURE TRENDS



(After Koff, 1991)

PROPULSIVE EFFICIENCY

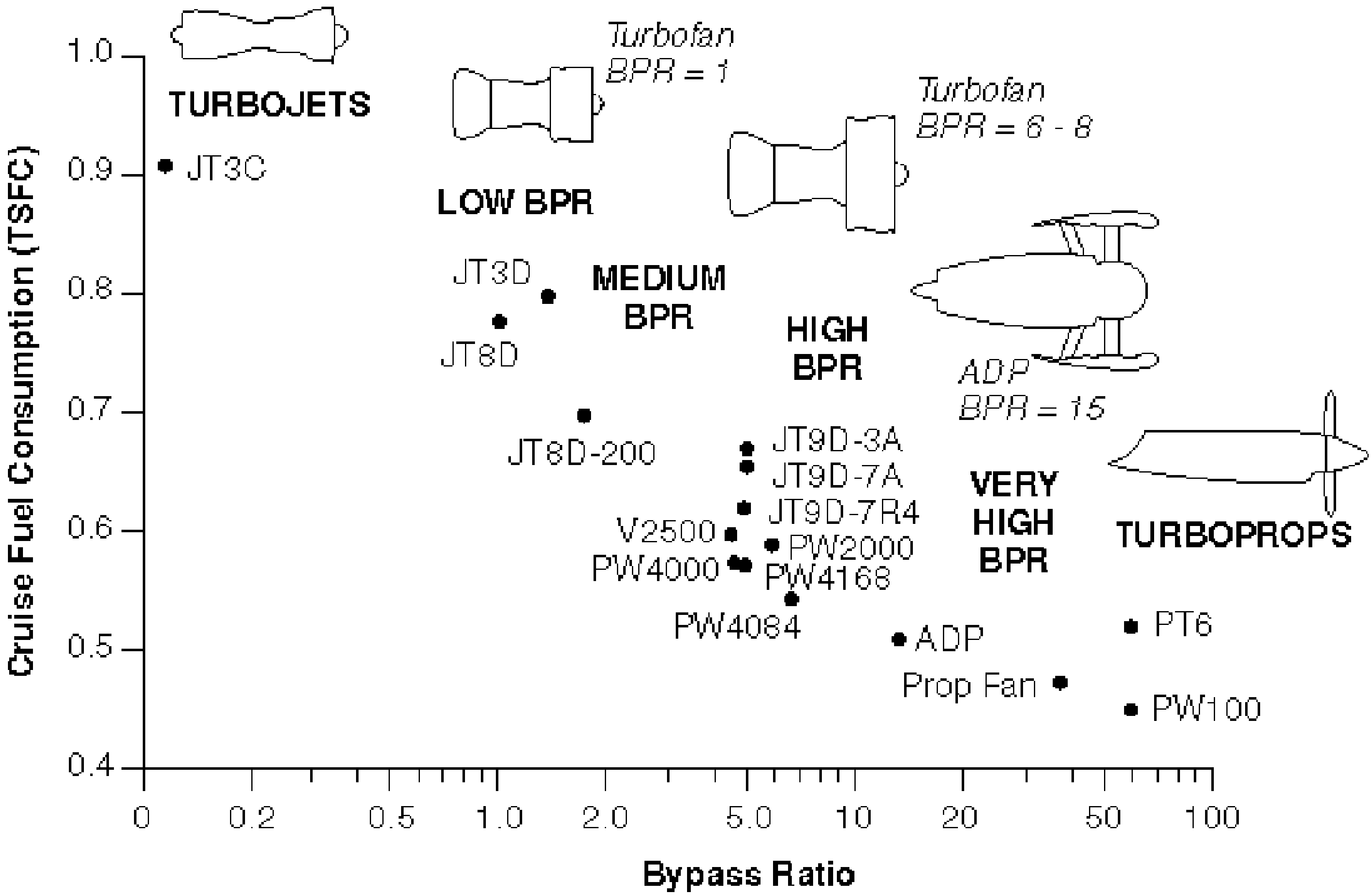
We can use our expression for thrust to rewrite the equation for propulsive efficiency in a more convenient form

$$T \approx \dot{m}\left(u_e - u_o\right) \qquad \left(\text{since } \dot{m}_e \approx \dot{m}_o\right)$$

Then,

$$\eta_p = \frac{\dot{m}\, u_o\left(u_e - u_o\right)}{\frac{\dot{m}}{2}\left(u_e^2 - u_o^2\right)} = \frac{2u_o}{u_o + u_e} = \frac{2}{1 + \frac{u_e}{u_o}}$$

TRENDS IN BYPASS RATIO



(After Koff, 1991)

OTHER EXPRESSIONS FOR OVERALL EFFICIENCY

Specific Impulse (I or I_{sp}):

$$I = \frac{\text{thrust}}{\text{fuel weight flow rate}}$$

(units: seconds)

Thrust Specific Fuel Consumption (SFC or TSFC):

$$SFC = \frac{\text{mass flow rate of fuel}}{\text{thrust}}$$

(units: lbm/hr/lbf or kg/s/N)

IMPLICATIONS FOR ENGINE DESIGN

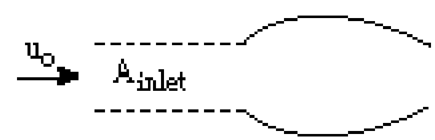
Considering jointly the expressions for thrust and propulsive efficiency,

$$F \cong \dot{m}(u_e - u_o) \qquad \eta_{prop} = \frac{2}{1 + \frac{u_e}{u_o}}$$

As $\frac{u_e}{u_o} \uparrow \quad \frac{F}{\dot{m}} \uparrow \quad \eta_{prop} \downarrow$

As $\frac{u_e}{u_o} \rightarrow 1 \quad \frac{F}{\dot{m}} \downarrow \quad \eta_{prop} \uparrow$

Also, as $\frac{F}{\dot{m} u_o} \uparrow \quad A_{inlet} \downarrow \quad \text{Drag} \downarrow$



PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

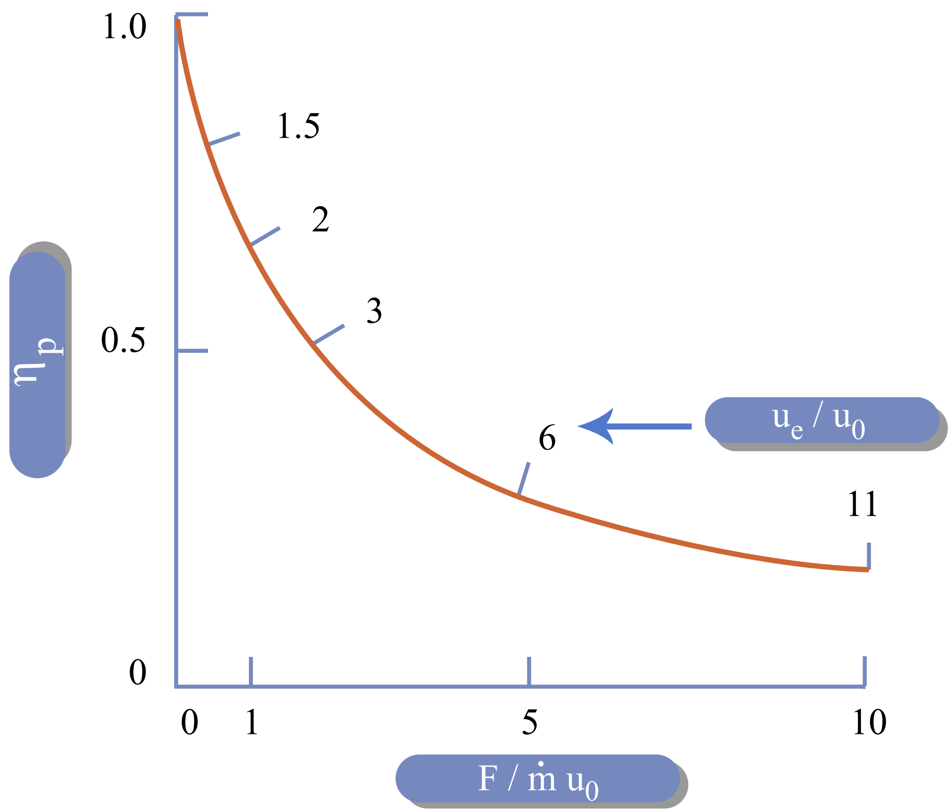
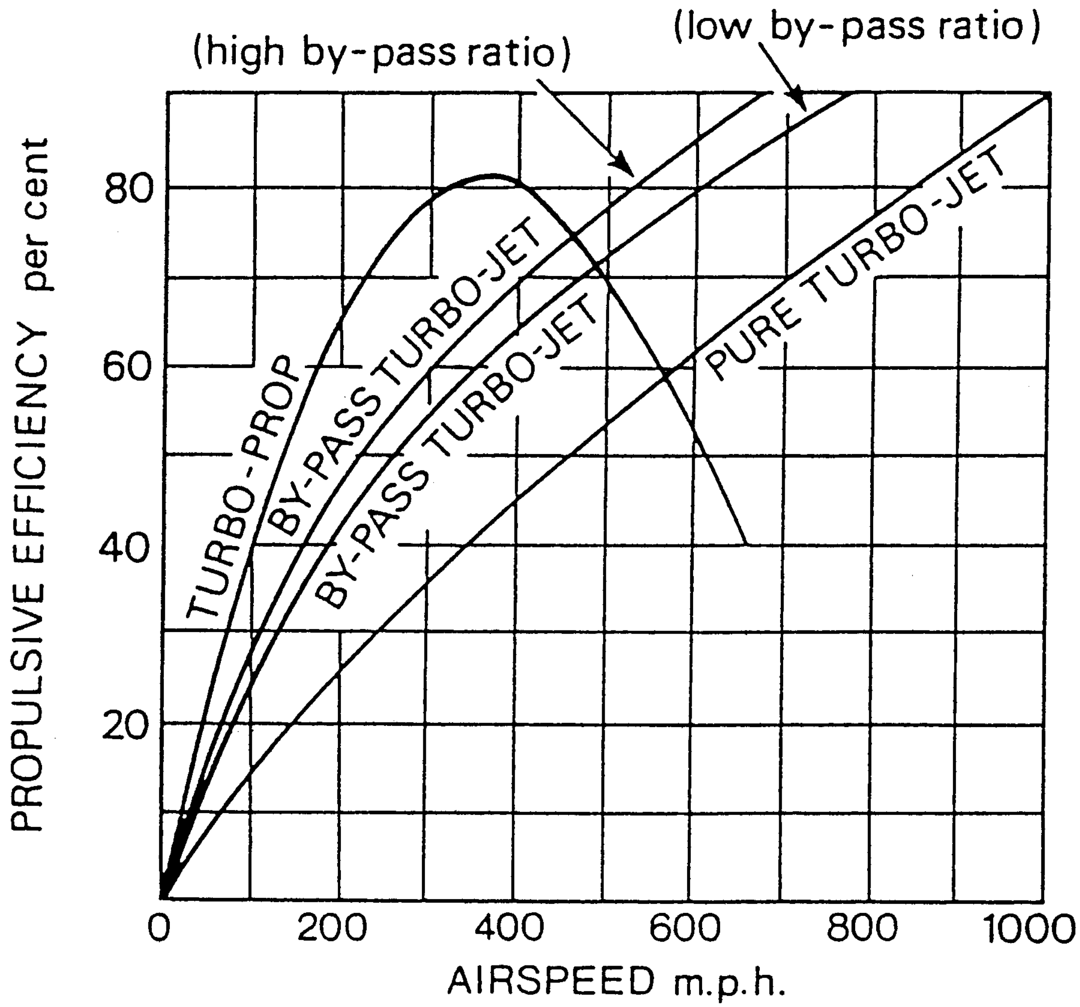


Image adapted from: Kerrebrock, J. L. *Aircraft Engines and Gas Turbines.* 1991.

PROPULSIVE EFFICIENCY VS. FLIGHT SPEED



PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

For fighter aircraft that need high thrust/weight and fly at high speed, it is typical to employ engines with smaller inlet areas and higher thrust per unit mass flow

However, transport aircraft that require higher efficiency and fly at lower speeds usually employ engines with relatively larger inlet areas and lower thrust per unit mass flow

PROPULSIVE EFFICIENCY

Image removed due to copyright considerations.

At low flight velocities, the highest propulsive efficiency is typically obtained with a propeller or an unducted fan

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AIRCRAFT AND ENGINE DESIGN ISSUES

- **Thrust ≈ (mass flow) x (change in velocity across engine)**

- **Range ~ fuel efficiency (commercial and military)**
  - **Thermal efficiency** — High pressures and temperatures
  - **Propulsive efficiency** — Large mass flow with small velocity change

- **Maneuverability (military)**
  - **High thrust-per-weight, small compact engine**

- **Supersonic flight (military)**
  - **Low drag, small compact engine**

(Maneuverability and supersonic flight:) High energy conversion per unit volume (high temperatures and pressures)

Small mass flow with large velocity change

Waitz, 2002

AN EXAMPLE OF CYCLE PERFORMANCE IMPROVEMENT