Flight Power Relations

9 Feb 06

Lab 1 Lecture Notes

Nomenclature

D	aircraft drag
L	aircraft lift
W	aircraft weight
T	propeller thrust
V	flight speed
S	reference area (wing area)
b	wing span
AR	wing aspect ratio
C_L	lift coefficient
C_D	drag coefficient
CDA_0	drag area of non-wing components
c_ℓ	wing-airfoil profile lift coefficient
c_d	wing-airfoil profile drag coefficient
ρ	air density
P	thrust power ($\equiv TV$)
P_{shaft}	motor shaft power
P_{elec}	electric power (Volts $\times$ Amps)
η_m	electric motor efficiency
η_p	overall propeller efficiency
η_{ideal}	ideal propeller efficiency
R	propeller radius
T_c	thrust coefficient
Ω_m	motor rotation rate
Ω	propeller rotation rate
λ	propeller advance ratio
Re	chord Reynolds number
E_{elec}	electrical (battery) energy
$t_{\max}$	maximum flight duration

Thrust Power

Generation of thrust during flight requires the expenditure of power. In steady level flight, $T = D$, and hence the thrust power is equal to the drag power.

$$P \;\equiv\; TV \;=\; DV \qquad\qquad \text{(steady level flight)} \tag{1}$$

In steady level flight we also have $W = L$, which gives the velocity in terms of other relevant parameters.

$$W \;=\; L \;=\; \frac{1}{2}\rho V^2 \, S \, C_L \tag{2}$$

$$V \;=\; \left(\frac{2\,W}{\rho\, S\, C_L}\right)^{1/2} \tag{3}$$

The drag power can then be given as follows.

$$DV \;=\; \frac{1}{2}\rho V^3 \, S \, C_D \tag{4}$$

$$DV \;=\; \left(\frac{2W^3}{\rho\, S}\right)^{1/2} \frac{C_D}{C_L^{3/2}} \tag{5}$$

We will assume that the typical wing airfoil sees the same local c_ℓ as the overall aircraft C_L, so we can employ 2D airfoil $c_d(c_\ell, \mathit{Re})$ data.

$$c_\ell \;=\; C_L \tag{6}$$

The aircraft drag coefficient can now be broken down into three basic components.

$$C_D \;=\; \frac{\mathit{CDA}_0}{S} \;+\; c_d(C_L, \mathit{Re}) \;+\; \frac{C_L^2}{\pi\, \mathit{AR}} \tag{7}$$

The last term is the induced drag, which directly depends on the aspect ratio of the wing. This is defined in terms of the wing span and area.

$$AR = \frac{b^2}{S} \tag{8}$$

Figure 1 shows the three C_D components versus C_L for a typical 1.5 m span light RC sport aircraft.

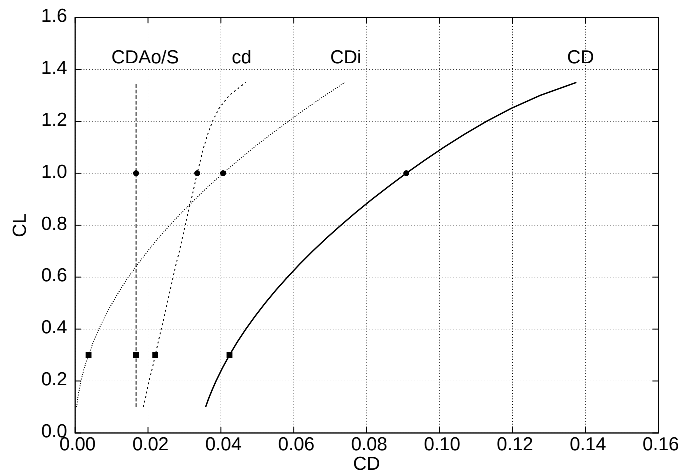

Figure 1: Drag polar and drag polar components for electric sport aircraft. $AR = 9.0$

For a typical operating point at $C_L = 1.0$ (low speed) and $C_L = 0.3$ (high speed), indicated by the symbols in Figure 1, the three components contribute roughly the following percentages to the total drag:

C_L	CDA_0/S	c_d	$C_L^2/\pi AR$	C_D
1.0	0.0167	0.0335	0.0406	0.0909
0.3	0.0167	0.0220	0.0037	0.0424
1.0	18 %	37 %	45 %	100 %
0.3	39 %	52 %	9 %	100 %

The corresponding flight power is shown in Figure 2.

Flight Power and Duration

In an electric aircraft, the flight power is provided by an electric motor, driving a propeller with some efficiency η_p.

$$P = \eta_p \, P_{\text{shaft}} \tag{9}$$

The motor itself has efficiency η_m, and is supplied by a battery which outputs electrical power P_{elec}.

$$P_{\text{shaft}} = \eta_m \, P_{\text{elec}} \tag{10}$$

$$P = \eta_p \, \eta_m \, P_{\text{elec}} \tag{11}$$

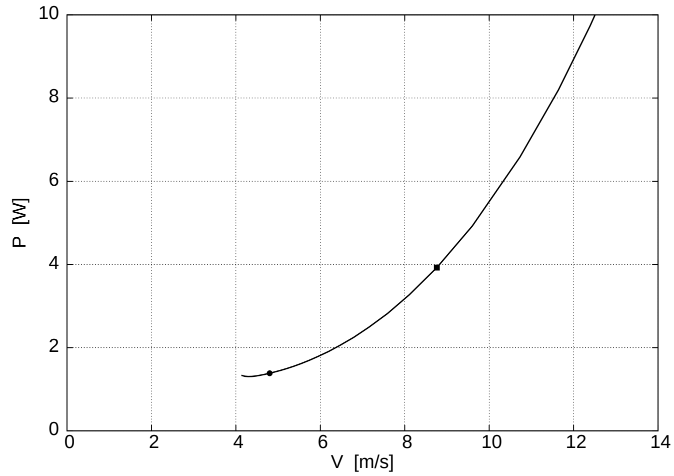

Figure 2: Thrust power $P = DV$ for electric sport aircraft.

Combining the relations above, the electrical power required for level flight is given by the following relation:

$$P_{\text{elec}} \;=\; \frac{1}{\eta_p\,\eta_m}\,P \;=\; \frac{1}{\eta_p\,\eta_m}\left(\frac{2W^3}{\rho\,S}\right)^{1/2}\left(\frac{CDA_0/S}{C_L^{3/2}} + \frac{c_d}{C_L^{3/2}} + \frac{C_L^{1/2}}{\pi\,AR}\right) \tag{12}$$

For a given available battery energy E_{elec}, the maximum flight duration is then inversely proportional to the minimum possible electrical power needed to sustain flight.

$$t_{\max} \;=\; \frac{E_{\text{elec}}}{(P_{\text{elec}})_{\min}} \tag{13}$$

Hence, for a fixed amount of battery energy, the maximum duration is obtained by minimizing P_{elec}. As suggested by Figure 2, this minimum power typically occurs close to the minimum possible flight speed just short of stall.

To obtain maximum speed, it is clearly necessary to use the maximum available electrical power. The maximum speed (or minimum C_L), is then implicitly determined by equation (12).

$$P_{\text{elec}} \;=\; (P_{\text{elec}})_{\max} \qquad \rightarrow \qquad C_{L_{\min}}, V_{\max} \tag{14}$$

Parameter Coupling and Design Optimization

It's essential to realize that most of the variables in equation (12) are coupled in an actual design application. So that when one design parameter is changed, its effects on equation (12) can enter in a number of ways, not just via its explicit appearance. Two examples which might appear if one attempts to decrease $(P_{\text{elec}})_{\min}$:

- Increase the wing area S
  - Pro: Direct $1/S^{1/2}$ reduction of P_{elec}, direct $1/S$ reduction of the CDA_0 term
  - Con: Increases the aircraft's weight W because of more wing material
  - Con: May require increasing W even more for adequate strength

- Con: Reduces V, which increases T_c, which decreases η_{ideal}, which decreases η_p

- Use more efficient motor, with larger η_m.
  - Pro: Direct $1/\eta_m$ reduction of P_{elec}
  - Con: More efficient motor may be heavier, and hence may increase W.

Other Pros and Cons may be present in addition to those listed above, depending on the situation.

Much of the activity which occurs during aircraft design and sizing consists of identifying and quantifying such couplings. Knowing the couplings then allows suitable tradeoffs to be performed, in order to find the best set of design parameters to maximize the design objective. Once a good or optimum design has been reached, all its competing tradeoffs are in balance, so that there are no more "easy" design changes which can be made without adversely affecting something else.