Lecture S8 Muddiest Points

General Comments

Today, we did a little review of Laplace transforms, and saw how to use them in the analysis of systems. The most confusion seems to be about the region of convergence.

Responses to Muddiest-Part-of-the-Lecture Cards

(17 cards)

1. ***You always want us to out the region of convergence, but what does it mean to converge or not converge? (1 student)*** Consider the LT of $g(t) = e^{at}\sigma(t)$. The LT integral is given by
$$G(s) = \int_0^\infty e^{at}e^{-st}\,dt = \int_0^\infty e^{(a-s)t}\,dt$$
As $t \to \infty$, the integrand either goes to zero or goes to infinity, depending on whether $a - s$ is negative or positive. If $a - s$ is negative, the integrand goes to zero exponentially fast, which means the integral is finite (there is finite are under the graph of $e^{(a-s)t}$), so we say the integral converges. if $a - s$ is positive, the integrand blows up, so the integral is infinite — it doesn't converge. So the LT is only well-defined for $s > a$.

2. ***Could you explain more clearly what the [pole-zero and region of convergence plot] means? (1)*** The LT integral converges for some values of s. The hashed region in the plot represents the complex values of s for which the integral converges. The x's on the plot represent the poles of the system. The o's on the plot represent the zeros of the system.

3. ***Confused as to why*** $\mathrm{Re}[s] > 0$ ***in most cases. It seems to have something to do with the value in the exponential. (1)*** For unilateral LTs, the region of convergence is always of the form $\mathrm{Re}[s] > \sigma_0$, for some σ_0. The reason is that if the integral converges for some value of s, the integrand of the LT integral will get smaller if the real part of s is increased. That is, the factor e^{-st} gets smaller as s gets more positive.

4. ***How do you write the region of convergence for***
$$G(s) = \mathcal{L}\left[e^{-at}\right] = \frac{1}{s+a}$$
Should it be $s > a$ ***or*** $s > -a$***? (1)*** $s > -a$. The pole of $G(s)$ is where the denominator is zero, namely, at $s = -a$. The region of convergence is to the right of the pole.

5. ***What's a pole? (1)*** The pole of a transfer function $G(s)$ are those values of s for which $G(s) = \infty$. This happens whenever the denominator of $G(s)$ is zero. Actually, my definition of a pole above is too sloppy to be correct, but will do for now.

6. ***Often, the differential equations don't come with initial values. Should we just solve with them unknown? (1)*** No. Usually, there will be a specific right thing to do, and it won't involve solving with unknown ICs. For example, when you find the impulse response, the ICs are implicitly zero.

7. ***For a system with impulse response $g(t) = \sigma(t)$, how does that lead to***

$$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$$

(1) The response to arbitrary input $u(t)$ is

$$y(t) = g(t) * u(t) = \int_{-\infty}^{\infty} \sigma(t-\tau)u(\tau)\,d\tau = \int_{-\infty}^{t} u(\tau)\,d\tau$$

since $\sigma(t-\tau) = 1$ for $\tau < t$.

8. ***Why is it that the integral***

$$\int g$$

sometimes instead of (from last lecture)

$$\int g'$$

(1) I have to admit that I don't understand your question. Please ask again at next lecture, or at the recitation.

9. ***Please do some real examples (not RLC circuits). (1)*** Although RLC circuits are real examples, I take your point, and will try.

10. ***No mud. (8)***